GSI-92-24
REPORT
OKTOBER 1992
ISSN 0171-4546

DESIGN OF A LIGHT ION MEDICAL SYNCHROTRON

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Design of a Light Ion Medical Synchrotron

Introduction

As it has been described in reference [1], it is necessary to design a synchrotron facility for the requirement of cancer therapy.

The sketch of the dose deposition versus depth in tissue (equivalent to water) was given in Fig. A. From the figure, the conventional radiation of Bremsstrahlung, γ-ray and neutrons shows the undesirable decay of energy deposition with progressing penetration. The dose applied to a deep seated tumor is frequently much smaller than the dose to the healthy tissue.

A large fraction of the localized tumors which are seated close to the sensitive tissue, are not curable by surgery or classical radiotherapy, but they have been treated successfully by proton beams which were initially available from the old cyclotrons or synchro-cyclotrons. Because of the sharp Bragg peak for the protrons and light ions as shown in Fig. A, and for light ions the enhanced relative biological effectiveness (RBE), ions will be more favourable compared to conventional radiation.

In most cases the tumor volume extends over a much larger depth range than the sharp Bragg peak and one must shift this peak slice by slice by adjusting the energy of ion beam step by step, and the dose applied to the healthy tissue in front and behind tumor volume will be expected to be minimized during the treatment. In other words, the accelerator with variable energy will be desired for cancer therapy.

A much larger number of cyclotrons are used for neutron therapy or isotope production. The compactness in size and the intensity are certainly the merit for the cyclotron type, but the disadvantage of the cyclotron is that it cannot be built with variable energy in the relativistic energy region, above 100 MeV/A for example, because the isochronous condition requires a strong shaping of the magnetic field, which cannot adequatly be obtained by trim coils, and the isochronous field condition could not be quickly retuned to the desired different energy as well.

An energy of up to 250 MeV/A is needed for the irradiation of deep seated tumors with protons. In case of a proton cyclotron, the absorber devices could be used to adjust and vary the energy in order to control the beam deposition. This method disturbes the optical sharpness of the proton beams and produces a mess of neutrons. As for the light ion beams the large quantities of fragments with different Z-values would be produced by the absorber and tend to dilute the distal edge of the Bragg peak.

Therefore the option of a light ion cyclotron, with its inherently fixed beam energy, is not recommendable.

A synchrotron can be freely and rapidly adjusted in energy. Extended experience in construction and operation of these machines have been established in the last four decades. In particular, the energy variation from cycle to cycle and the accordingly necessary readjustment of the beam line magnets have been demonstrated at GSI since 1991.

Requirements

The study of a medical light ion synchrotron, performed at GSI in the years 1990 - 1991, particularly aims at a very compact and simple design, fulfilling the requirements of a hospital based facility.

A separated function strong focussing optical lattice has been adopted from the beginning and the design options for components have been set on the conservative side. Engineering experience, gained during the SIS/ESR construction, have been included. But the high degree of performance flexibility, typical for a nuclear physics machine, was not included in the design goals.

The main targets for the accelerator designed are summarized as follows:

Ion type:	Fully stripped ions Ne^{10+}
Ion energy:	480 MeV/u
Ion intensity:	$> 1.0 \times 10^{8}$ pps
Beam emittance (normalized):	0.5 π mm-mrad

Neon ions were chosen as reference particle. Of course, lighter ions can be accelerated, as well. However, if biological experiments, performed at GSI, continue to substantiate the evidence that ions heavier than Carbon are not substantially more favourable, the machine parameters should be scaled down to Carbon ions as the heaviest particle.

A final energy of approximately 480 MeV/A is required to provide about 22 cm of the penetration depth in tissue for Neon ions as shown in Fig. B.

The 10^{8} pps of beam intensity which is three orders of magnitude lower than in nuclear physics machines, is based on the requirement of irradiating a one liter tumor volume in about 2 minutes. This requirement of lower beam intensity will relax the design constraints of a synchroron, and the single turn injection scheme which will drastically reduce the aperture requirement of the ring and save the costs, therefore is taken into account.

The medical accelerator consists of an ECR source, a RFQ structure and an IH linac which accelerate the Ne^{6+} to the desired injection energy. Then fully stripped Ne^{10+} ions will be injected in the separated function synchrotron which accelerates the ions to the desired final energy.

Main Characteristics of the Synchrotron

Injection Energy

Which injection energy is recommendable for the medical synchrotron? 5 MeV/A for the light ions is the answer when high stripping efficiency to the naked nucleus is aimed at. If the injection energy is reduced to 3 MeV/A it would largely decrease the length of Linac and evidently reduce the costs of the whole facility construction. At 3 MeV/A, the fully-stripped efficiency will be 53% rather than 83% at 5 MeV/A of injection energy. Some parameters such as the beam loss due to residual gas collisions, field imperfections at the start of the magnetic ramp and Rf swing etc, will become serious when the injection energy decrease to the 3 MeV/A. But it is pointed out in reference [2], according to the experiences of SIS construction in GSI, no technical penalty was found except the beam intensity will go down by a factor of 1.6, therefore, there is no safety factor left over for the ion source performance. One of the better ways is that the intensity improvement of the ion source of one order magnitude higher than the intensity of existing ones is expected.

The machine will still be operated under the space charge limit because of the admitted low beam intensity.

Lattice

For the arrangement of the focusing quadrupoles in multiplets there are basically three ways which are quite different in their optical properties.

The most common is the FODO scheme which requires the minimum focusing strength but at the same time it has the largest variation in amplitude functions. A factor of 2 between minimum and maximum is typical and is equivalent to about a factor of 1.5 in beam diameter.

Just opposite in properties is the TRIPLET scheme, which provides very smooth and low amplitude functions but needs at least twice as much focusing strength for the same tune.

The DOUBLET scheme is very much like the FODO, only the space between F and D is different from that between D and F. This gives lower amplitudes than FODO but needs higher focusing fields depending on the ratio of distances.

Looking for the optimal arrangement for the synchrotron, one first try to minimize the necessary apertures in the dipole magnets, especially the vertical one because the costs are most sensitive to it. Since the aperture is designed for the maximum of the amplitude function there is some wasted space for the case of FODO lattice, the TRIPLET lattice is a better choice which gives the dipole gap fitting the average beam size optimally in every part. But one has to invest more in the aspect of stronger quadrupoles which can considerablly compensate the saving by the dipole gap reduction.

The correcting elements such as sextupoles and fast small quadrupoles etc, always affect both transversal motion of particles, the larger the difference of amplitude varition is, the easier one can correct the orbit using these correcting elements. It is obviously the merit for the case of the slow extraction. The FODO provides a more economic way in dealing with this kind of problem.

This is why most synchrotron are designed with FODO and similar lattices but only few use a TRIPLET structure for focusing. Based on the reasons mentioned above, the FODO or DOUBLET structure with its high amplitude at several places was preferred to be adopted in most of the lattice designs and also in the design of our medical synchrotron.

Because the DOUBLET lattice can further reduce the synchrotron size and keep the stronger focusing, it seems that the DOUBLET lattice would be a better choice for the medical synchrotron design.

The assumed global parameters at the exit of the injector are:

Energy		Mev/A	3
Particle intensity during pulse		pps	2.6×10^{13}
Repetition rate		Hz	1
Beam emittance	(ε_h and ε_v)	π mm-mrad	6.2
Momentum spread	($\Delta p/p$)		0.3%

For the magnetic peak field of the dipoles a rather moderate value of 1.4 Tesla is assumed. It guarantees good field quality and low operation costs. In our case, the curvature radius of 5 m with 1.4 Telsa of the maximum field strength for the dipoles is assumed to achieve 7.07 Tm of the beam rigidity needed.

In order to reduce the size of the synchrotron, only four long straight sections (lss) are needed for the installation of the injection elements, the elements for the resonance extraction, rf cavity, some auxiliary elements used for the slow extraction such as sextupoles, fast quadrupoles for the tune shift and bumping magnets etc. Therefore a 4-fold symmetry with DOUBLET lattice is described and calculated in optical properties. A 6-fold symmetry with DOUBLET lattice is simultanously described here as comparison, because it has more long straight sections which can accommodate more elements if needed and the direction of the injection and extraction can be selected optimally according to the condition of the site.

The main lattice parameters for 4-fold and 6-fold DOUBLET are summarized in Table 1, the tune diagram, the corresponding β-function in both direction and the dispersion function etc. are shown in Fig. 1.2 - 1.6 for the 4-fold case and Fig. 2.2 - 2.6 for the 6-fold case respectively.

The studies of the ion optics, described in this report have been performed with the help of some computer programs in GSI, which are:
- The MIRKO program, version 5.3
- The MULTITUR program for calculating the multiturn injection.

Table 1 Main Parameters of Medical Synchrotron

		4-fold	6-fold
1. Basic parameters			
Accelerated particle		Ne	
The ratio of mass to charge	A/q	20/10 = 2	
Injection energy Ei	(Mev/A)	3	
End energy Ef	(Mev/A)	480	
Maximum magnetic rigidity	(Tm)	7.0747	
2. Ring construction			
Circumference	(m)	54.616	62.916
Av. trajectory radius	(m)	8.69	10.01
Curvature radius of dipole	(m)	5.00	
Length of a focusing period (FP)	(m)	13.654	10.486
Number of FP's		4	6
FP structure		DOUBLET	
Number of dipoles		12	
- edge angle -	(deg)	0	
Number of quadrupoles		8	12
Length	(m)		
- dipole/quadrupole		2.618/0.45	
- drift space		0.50/2.90	
3. Beam characteristics			
		hori./vert.	*hori./vert.*
Tunes		1.648/1.412	2.316/2.227
Phase adv. per FP		148.3/127.1	139.0/133.6
transition energy		1.7398	2.6195
Maximum beam half-width	(mm)		
Betatron oscillation			
- quadrupoles		8.66/12.27	9.53/11.63
- dipoles		7.70/11.21	6.59/10.28
Dispersion			
- quadrupoles		14.85/	9.69/
- dipoles		11.31/	5.94/
Closed orbit distortion			
- quadrupoles		8.32/5.79	9.79/8.56
- dipoles		7.40/5.48	6.77/7.88
Total			
- quadrupoles		31.83/18.06	29.0/20.0
- dipoles		26.41/16.69	19.3/18.2
Vacuum chamber half aperture			
- quadrupoles	(mm)	45/45	
- dipoles	(mm)	40/20	
Beta function	(m)		
maximum		12.04/24.19	14.55/21.73
minimum		2.95/3.15	2.27/2.02
Dispersion	(m)	4.60/	3.23/
Natural chromaticity		-0.658/-1.668	-1.124/-2.049

		4-fold	6-fold
		hori./vert.	*hori./vert.*
Acceptance (without dispersion)	(πmm-mrad)	168.2/18.7	138.8/21.7
Acceptance (with dispersion)	(πmm-mrad)	76.1/18.8	86.1/21.4
Normalized emittance	(πmm-mrad)	0.5	
Emittance at injection	(πmm-mrad)	6.225	
Emittance at extraction	(πmm-mrad)	0.44	
Momentum spread at injection		± 0.003	

4. Magnets

Number of dipole in the ring		12	
Magnetic length of a diole	(m)	2.618	
Bending angle per magnet	(deg.)	30	
Aperture of the vacuum chamber	(mm^2)	80×40	
Minimum field (at injection)	(T)	0.1	
Maximum field	(T)	1.415	
Maximum field ramp	(T/s)	4.4	
Number of quadrupoles (QP)		8	12
Magnetic length of each QP	(m)	0.45	
Maximum field gradient	(T/m)	8.5	10.5
Diameter of the QP aperture	(mm)	90	
Usable field region	(mm)	100	

5. High Frequency Acceleration

Revolution frequency of particle	(MHz)	0.44-4.13	0.38-3.58
Acceleration field frequency	(MHz)	0.44-4.13	0.38-3.58
Harmonic number		1	
Eff. Accel. Voltage per Turn	(kV)	0.60	0.69
Phase angle	(deg)	45	
Number of Accelerating cavities		1	
Rf peak voltage per cavity	(kV)	0.85	1.0

6. Vacuum system

Average chamber pressure	(mbar)	1×10^{-9}	

Injection and Extraction

The yield of ECR source for the Ne^{6+} is assumed as about 33 PμA, it corresponds to 2.1×10^{14} pps, after the accelerating of a RFQ and a small Linac, the Ne^{+6} beam pass through a stripper which will strip fully the electrons outside the Ne nuclei with 34% of stripping efficiency and the beam intensity becomes about 5×10^{13} pps, then the beam is injected in into the synchrotron in single turn injection scheme in the revolution time of 2×10^{-6} which results in 1×10^{8} particles per cycle. In order to insure the intensity requirement of 1×10^{8} at the target the beam intensity at the exit of ECR ion source should be increased to 3 times as that extracted from the existing ion source which is mentioned above.

The intensity schedule for Ne beam is listed in Table 2 [3], the values on the Table are estimated without optimum performance of all system.

Table 2 Intensity Schedule of Medical Facility for Ne beam

	Loss Factor	Single-turn injection	Multi-turn injection
ion source		2.1×10^{14}	
Beam transport	0.9	1.9×10^{14}	
RFQ	0.8	1.5×10^{14}	
Linac 3 MeV/A	0.9	1.4×10^{14}	
Stripper	0.34	4.6×10^{13}	
Beam transport	0.9	4.2×10^{13}	
Inj. Single turn	2×10^{-6} (*)		
	0.9×10^{8}		
Inj. Multi turn	17 turns		1.5×10^{9}
	60%		9.2×10^{8}
RF capture	0.9	0.8×10^{8}	8.4×10^{8}
Sync. Rep. Rate	1.0	0.8×10^{8}	8.4×10^{8}
Accel. Eff.	0.9	0.7×10^{8}	7.5×10^{8}
Extract.	0.5	0.4×10^{8}	3.8×10^{8}
Beam transport	0.9	0.3×10^{8}	3.4×10^{8}

If the intensity improvement of ECR source is difficult to realize, it seems that the multi-turn injection will have to be applied, the injection of 15-20 turns number with $> 60\%$ of injection efficiency will be suitable for the desired intensity design, but the horizontal aperture of the dipoles has unavoidably to be enlarged.

It depends completely on the improvement of ECR source to decide which injection scheme will be adopted in the design. Since the possibility of intensity improvement is still open in the near future the two injection scheme are described and calculated alternatively in this report.

Single turn injection:

A single turn injection scheme of the medical synchrotron has been studied, resulting in the minimum transverse emittance of the circulating beam. The adoption of single-turn injection has a positive impact on several parameters including the injection efficiency, the synchrotron aperture, the extraction efficiency and the injector intensity requirement.

(*) The revolution period of Ne at 3 Mev/A of injection energy

This conventional single turn injection scheme consists of two elements: A magnetic inflector deflect the beam to the closed orbit with a small angle. Then a fast kicker magnet or an pulsed electrostatical deflector puts the beam straight on the central orbit. When the head of the injected beam portion is nearing the kicker position after one revolution, the kicker ist switched to zero.

Multi-turn injection:

Bumping magnets which are located upstream and downstream from the inflector section, distort the closed orbit and the distorted orbit can come back to original closed orbit within about 15 to 20 turns of the first beam injected, the field falling time will at most last dozens micro-second. The horizontal acceptance of the machine must be increased and the corresponding aperture is enlarged as well. In the injection section, as the same as the single turn injection there is an inflector magnet deflecting the beam and an electrostatic septum instead of the kicker magnet in the case of single turn injection guiding the beam onto the distorted orbit. The beam intensity can be increased to 10 times of the case of single turn injection, but there are more particles which will be lost on the wall of the elements.

Extraction:

A resonant extraction method can extract a long duration beam with high efficiency.

A classical third resonant extraction is applied in the design, a fast quadrupole shift the horizontal Q-value toward the resonance point, a sextupole push the beam to the unstable region, an electrostatic septum, the radial and angular position of which inadjustable, deflects the beam to the two extraction magetic septa which are located at the position of about 270^{o} or 90^{o} of the phase advance with respect to the electrostatic septum.

The parameters of injection including the single-turn injection and multi-turn injection, and resonance extraction are listed in Tables 3 - 5 for the cases of 4-fold and 6-fold lattices and the corresponding diagrams of beam envelopes are illustrated in Fig. 10 - 19 respectively.

Table 3 Parameters of Single-Turn Injection

<table>
<tr><th></th><th></th><th>4-fold</th><th>6-fold</th></tr>
<tr><td>1. General Characteristics</td><td></td><td></td><td></td></tr>
<tr><td>Ion energy at injection</td><td>(Mev/A)</td><td colspan="2">3</td></tr>
<tr><td>Revolution frequency</td><td>(MHz)</td><td>0.4395</td><td>0.3815</td></tr>
<tr><td>Transverse emittance</td><td>(πmm-mrad)</td><td colspan="2">6.225</td></tr>
<tr><td>2. Inflector magnet</td><td></td><td></td><td></td></tr>
<tr><td>Magnetic rigidity</td><td>(Tm)</td><td colspan="2">0.50</td></tr>
<tr><td>Deflection</td><td>(mrad)</td><td colspan="2">210</td></tr>
<tr><td>Field</td><td>(T)</td><td colspan="2">0.105</td></tr>
<tr><td>Eff. length</td><td>(m)</td><td colspan="2">1.0</td></tr>
<tr><td>Gap dimension</td><td>(mm^2)</td><td colspan="2">80×40</td></tr>
<tr><td>$N \times I$</td><td>(kA)</td><td>3.34</td><td>3.2</td></tr>
<tr><td>3. Kicker</td><td></td><td></td><td></td></tr>
<tr><td>Magnetic rigidity</td><td>(Tm)</td><td colspan="2">0.50</td></tr>
<tr><td>Deflection</td><td>(mrad)</td><td colspan="2">60</td></tr>
<tr><td>Field</td><td>(T)</td><td colspan="2">0.037</td></tr>
<tr><td>Eff. length</td><td>(m)</td><td colspan="2">0.8</td></tr>
<tr><td>Gap dimension</td><td>(mm^2)</td><td colspan="2">80×40</td></tr>
<tr><td>$B \times L$</td><td>($T \times m$)</td><td colspan="2">0.03</td></tr>
<tr><td>Fall time</td><td>(ns)</td><td colspan="2">320</td></tr>
<tr><td>Number of subunits</td><td></td><td colspan="2">2</td></tr>
<tr><td>Voltage p. subunit</td><td>(kV)</td><td colspan="2">30</td></tr>
<tr><td>Max. current</td><td>(kA)</td><td colspan="2">1.2</td></tr>
<tr><td>Inductance per subunit</td><td>(μH)</td><td colspan="2">1.0</td></tr>
</table>

Table 4 Parameters of Slow Extraction

		4-fold		6-fold
1. General Characteristics				
Ion energy at extraction	(Mev/A)		480	
Revolution frequency	(MHz)	4.13		3.58
transverse emittance	(πmm-mrad)		0.44	
horizontal tune		5/3		7/3
2. Electrostatic septum				
Electro. rigidity	(MV)		1600	
Deflection	(mrad)		5	
Field	(kV/cm)		70	
Length	(m)		1.2	
Gap width	(mm)		20	
Septum type			0.1 mm wires	
3. Magnetic sepıa				
Type			*septum 1/septum 2*	
Deflection	(mrad)		35/95	
Field	(T)		0.50/0.84	
Eff. length	(m)		0.5/0.8	
Gap dimension	(mm^2)		50 × 40/50 × 40	
Septum width (eff.)	(mm)		10/20	
N × I	(kA)		16.0/27.0	
4. Bumping magnets				
Number of units			2	
Max. Field	(T)		0.12	
Eff. length	(m)		0.3	
Gap dimension	(mm^2)		80 × 40	
N × I	(kA)		3.8	
5. Sextupole magnets				
Number of s. in ring			1	
Sextupole strength	(T/m)		9.	
Maximum d^2B/dx^2	(T/m^2)		30	
Eff. length	(m)		0.3	
Aperture diameter	(mm)		90	
6. Fast quadrupole for Q-shift				
Number of q. in ring			1	
Focusing strength	(T)		0.06	
Maximum field gradient	(T/m)		0.20	
Eff. length	(m)		0.3	
Aperture diameter	(mm)		90	

Table 5 Parameters of Multi-Turn Injection

		4-fold		6-fold
1. General Characteristics				
Ion energy at injection	(Mev/A)		3	
Revolution frequency	(MHz)	0.4395		0.3815
Transverse emittance	(πmm-mrad)		6.225	
Horizontal acceptance	(πmm-mrad)		100	
Vertical acceptance	(πmm-mrad)	18.5		21.2
Number of turn injected		16-17		15-16
Injection efficiency		60%		64%
Injected time	(μs)		35-45	
2. Inflector magnet				
Magnetic rigidity	(Tm)		0.50	
Deflection	(mrad)		210	
Field	(T)		0.105	
Eff. length	(m)		1.0	
Gap dimension	(mm^2)		80×40	
$N \times I$	(kA)		3.34	
3. Electrostatic Septum				
Electro. rigidity	(MV)		12.0	
Deflection	(mrad)		60	
Field	(kV/cm)		9.0	
Length	(m)		0.8	
Gap width	(mm)		20	
Septum type			0.1 mm wires	
4. Bumping magnets				
Number of units			3	
Max. field	(T)		0.025	
Eff. length	(m)		0.3	
Gap dimensions	(mm^2)		80×40	
$N \times I$	(kA)		0.8	
Drop time	(μs)		35-45	

Necessary Aperture

The required aperture for the beam injection is obtained by summing up the betatron oscillation amplitude of the injected beam, the space due to the momentum spread or dispersion of the beam, and the closed orbit distortion (COD) [5]. The former two terms are easily obtained from the optical calculation of the lattice.

In synchrotron, because the magnets cannot be made absolutely identical as ideal ones the field error $\delta(Bl)$ which can be expressed by

$$\delta(Bl) = \int Bdl - (\int Bdl)_{ideal} \tag{1}$$

will cause closed orbit distortion (COD).

The other field imperfection, such as survey error is also equivalent to field error and all of these field errors are regarded as a random distribution around the ring.

The important consideration for us in designing a lattice is to keep the closed orbit distortion to a minimum, the aperture will, therfore be reduced as much as possible to save power consumption and reduce the machine cost.

As the first step of consideration, that is how to decide the closed orbit distortion when the field error is given. A short dipole, which is assumed to be a delta function at azimuthal point s, cause a constant angular kick in divergence

$$\delta y' = \delta(Bl)/(B\rho)$$

and it can produce a kink in the orbit of a zero emittance particle at the point of kick.

In estimating the effect of a random distribution of dipole errors we take the r.m.s. average, weighted according to the β_k values over all of the kicks δy_i from the N magnets in the ring.

The expectation value of maximum $y_p(s)$ of COD is given with p confidence level.

$$y_p(s) = k(p) \times [(1 + |\sin(\pi Q)|/3) \times 2]^{1/2} \times \langle y(s) \rangle \tag{2}$$

where $k(p)$ is 2.40 for $p = 98\%$ and the case of elliptical vacuum chamber, and

$\langle y(s) \rangle = \beta(s) \times \beta \times N/8^{1/2} \times (\Delta Bl)_{rms}/B\rho/\ \sin(\pi Q)$

$\beta(s)$: the value of β function at reference point s.

β: the average over all the ring.

Q : the tune of synchrotron

$(\Delta Bl)_{rms}$: the r.m.s. average, weighted according to the β_k values over all of δy_i from the N magnets in the ring.

$B\rho$: the magnetic rigidity of the ring.

The principal imperfections causing orbit distortion in a synchrotron are :

type of element	source of error	r.m.s value	$<\Delta Bl>_{rms}/B\rho$	plane
quadrupole	displacement	$<\Delta y>$	$kl<\delta y>$	x,z
dipole	tilt	$<\Delta\theta>$	$\theta_i<\Delta\theta>$	z
dipole	length error	$<\Delta L/L>$	$\theta_i<\Delta L/L>$	x
dipole	field error	$<\Delta B/B>$	$\theta_i<\Delta B/B>$	x
dipole	stray field	$<\Delta B_{inj}/B_{inj}>$	$\theta_i<\Delta B_{inj}/B_{inj}>$	x,z

θ_i: the bending angle of dipole.

The first line in the table represents the random variations in the position of quadrupole magnets with respect to their ideal location. A small displacement of a quadrupole gives an effective dipole perturbation, $kl\times\delta y$. Where k is the division of field gradient of quadrupole (B_0/a) by the magnetic regidity ($B\rho$) and l is the effective length of quadrupole. The tilt of bending magnets causes a small resultant dipole in the horizontal direction which deflects vertically. and the field error of bending magnets causes a orbit distortion in x direction.

The error in the length of bending magnet, ΔL, causes an error of deflection angle $<\Delta L/L>$. If a stray field of the strength ΔB_{inj} exists at the injection, the kick by this field is given like in the case of field error of bending magnet.

In the estimation of COD, the influences of the stray field in the straight section is neglected.

The r.m.s values of errors for medical synchrotron should be more serious than for the case of SIS at GSI due to its less sector number of bending magnets and larger β value at quadrupole and bending magnet.

The errors of magnetic elements are assumed and listed as follows:

(1) The displacement of quadrupole for both horizontal and vertical direction:	± 0.1 mm
(2) The tilt of bending magnet:	0.1 mrad
(3) The field error of bending magnet:	$\pm 2\times 10^{-4}$
(4) The length error of bending magnet:	± 0.5mm

The COD calculation results of medical synchrotron are listed as follows for 4-fold and 6-fold respectively.

Radial direction

	4-fold		6-fold	
	dipole	quad.	dipole	quad.
Field err. ($\Delta B/B$)	5.0 mm	5.6 mm	4.4 mm	6.3 mm
Length err. ($\Delta L/L$)	5.0 mm	5.6 mm	4.4 mm	6.3 mm
displ. err. Quad. (Δx)	2.1 mm	2.4 mm	2.8 mm	4.0 mm
rms sum	7.4 mm	8.3 mm	6.8 mm	9.8 mm

Vertical direction

	4-fold		6-fold	
	dipole	quad.	dipole	quad.
Tilt of bend. ($\Delta\theta$)	4.2 mm	4.4 mm	4.9 mm	5.3 mm
displ. err. Quad. (Δx)	3.5 mm	3.7 mm	6.2 mm	6.7 mm
rms sum	5.5 mm	5.8 mm	7.9 mm	8.6 mm

Associated with the values contributed by the betatron oscillation for the case of single turn injection and dispersion both for the 4-fold and 6-fold lattices, the maximum beam half-width in the dipoles and the quadrupoles are listed

	4-fold	6-fold
radial direction (bending magnet):	26.4 mm	19.3 mm
(quadrupoles):	31.8 mm	29.0 mm
vertical direction (bending magnet):	16.7 mm	18.2 mm
(quadrupoles):	18.1 mm	20.2 mm

Therefore we can design the values of half aperture in the single turn injection scheme as 40 mm (horizontal), 20 mm (vertical) for the dipoles and 45 mm (both direction) for the quadrupoles.

As for the multi-turn injection scheme, the emittance of injected beam will be enlarged more 10 times than the emittance of the beam before injection, and the efficiency of multi-turn injection is about 60% which is mentioned in sec. 2. According to the formula $a^2 = \beta\varepsilon$, the values of betatron amplitude for the multi-turn injection can be estimated, where a is the betatron amplitude and ε is the emittance of the beam.

When the emittance of injected beam is assumed to increase to 100 π mm-mrad for example, and the turn number of injection is about 16 to 18, the values of half aperture must be increased to 65 mm (quadrupoles) and 55 mm (horizontal in dipoles) for the case of 4-fold lattice and 65 mm (quadrupoles) and 45 mm (horizontal in dipoles) for the 6-fold case respectively.

RF Accelerating System

The Ne^{10+} ions are accelerated by the electromagnetic field when they pass through the resonant cavity located at long straight section of synchrotron.

The ratio of the revolution frequency at the maximum energy 480 MeV/A to that at the injection energy 3 MeV/A is about 9.3 according to the relation

$$f = v/(2\pi R) = \beta \times c/(2\pi R) = f_o \times \beta$$

where R is the mean radius of the synchrotron and c is the light velocity, respectively.

The value 9.3 of RF swing is higher than the value of 7 for SIS in GSI. A factor of 10 of swing value at Saturne II was realized without difficulties

The synchronous energy gain ΔE per turn is related to the change in the magnetic field B as

$$\Delta E = 2\pi\rho Rqe(dB/dt) = qeV \times \sin(\phi_s)$$

where q is the charge state of Ne ions. When the synchronous phase ϕ_s is chosen as 30^o and the ramping time of the megnetic field of the dipoles is asumed as 22. Tm/s the required RF voltage is about 1.5 kV for both case of designed lattice.

The area of separatrix in the (w,ϕ) space of the longitudinal motion is given by

$$A_{bs} = 8\beta \,[2eV_{RF}E_s/\ (h\pi\,\eta\ )\ ]^{1/2}$$

with

$$\eta = E_s/\omega_s \cdot d\omega/dE$$

$$= (1/\gamma_t^{\,2} - 1/\gamma^2)/\beta^2$$

where γ_t is the transition energy in unit of rest energy of nucleon, and the expression

$$(\Delta E/E_s) = \beta \times [eV/(\pi h\eta E_s) \times G(\phi_s)]^{1/2}$$

is called the RF acceptance. The function $G(\phi_s)$ is given by

$$G(\phi_s) = 2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s$$

The RF acceptance plays an important role when designing a machine since it determines the capture efficiency at injection.

The value of the transition energy calculated in the lattice design is larger than the particle energy during the acceleration in order to insure a larger rf capture efficiency and the accelerating phase unchange.

Acknowledgements.

The author is greatful to D. Böhne for fruitful discussions during the design study. The availability of the calculation progams and the warm help provided by B.Franczak and H.Eickhoff are acknowledged here.

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[6] B.Franczak, SIS Parameter List, GSI-SIS-TN /87-13, 1987

Post scriptum

After this report was terminated in spring 1992, the following parameter changes occured:

1. The missing factor of 3.3 in the intensity budget table 2 for the single turn injection can be made up as follows:
   a) Charge state 5^+ instead of 6^+ from the source; this brings a factor of 2.4.
   b) Linac and stripping energy 5 MeV/u instead of 3 MeV/u; this brings a factor of 1.6. The increase in linac potential was found affordable after the ordering process for the CERN lead injector components.
   c) The extraction improvements at the SIS meanwhile point in the direction that a 90 $\pm$ 5 % extraction efficiency can be expected routinely.

2. The routine operation of the ECR source at the high charge state injector at GSI repeated the evidence that literatur data on ion source yield and emittance are too much on the sunny side. Before deciding on the preferred single turn injection, actual source data have to be confirmed. The hardware is available at GSI.

September 1992, D. Böhne

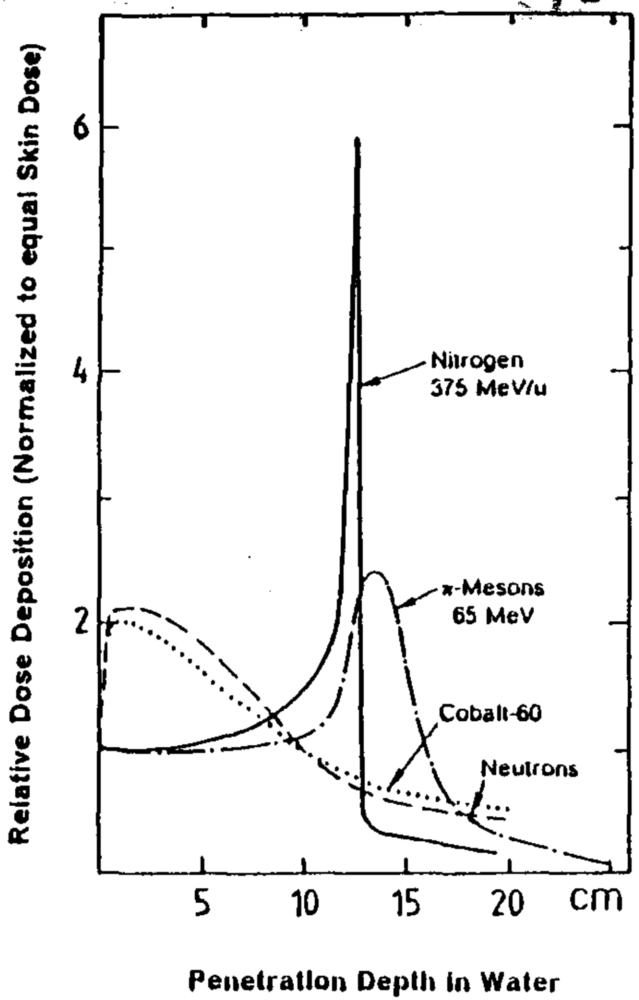


Fig. A Energy deposition of various ionizing beams in water or tissue.

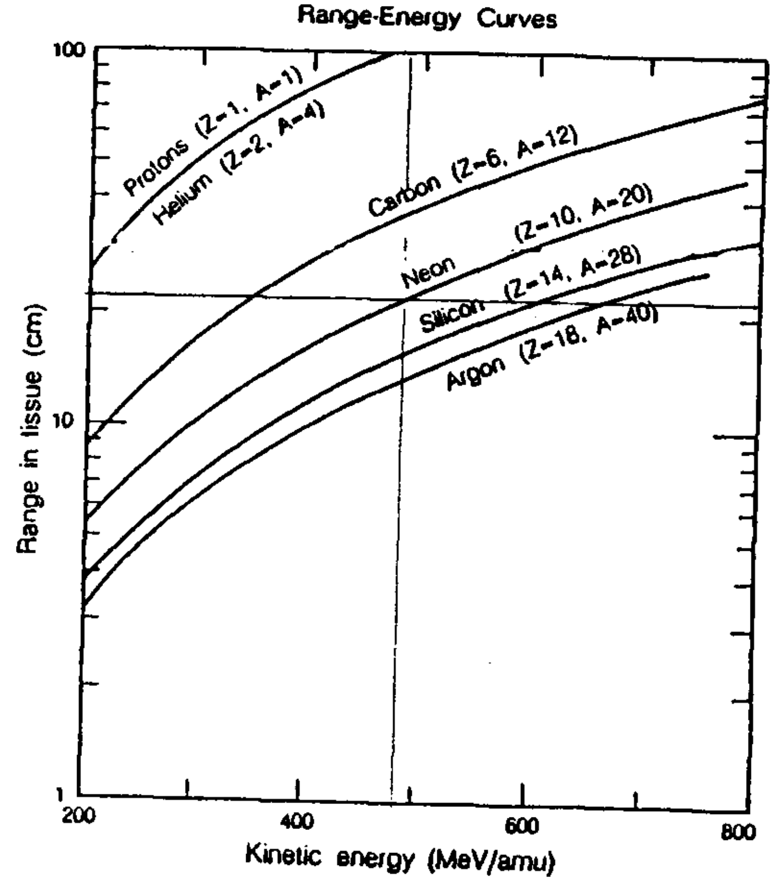


Fig. B Range-energy curves (penetration in tissue) for various ions.

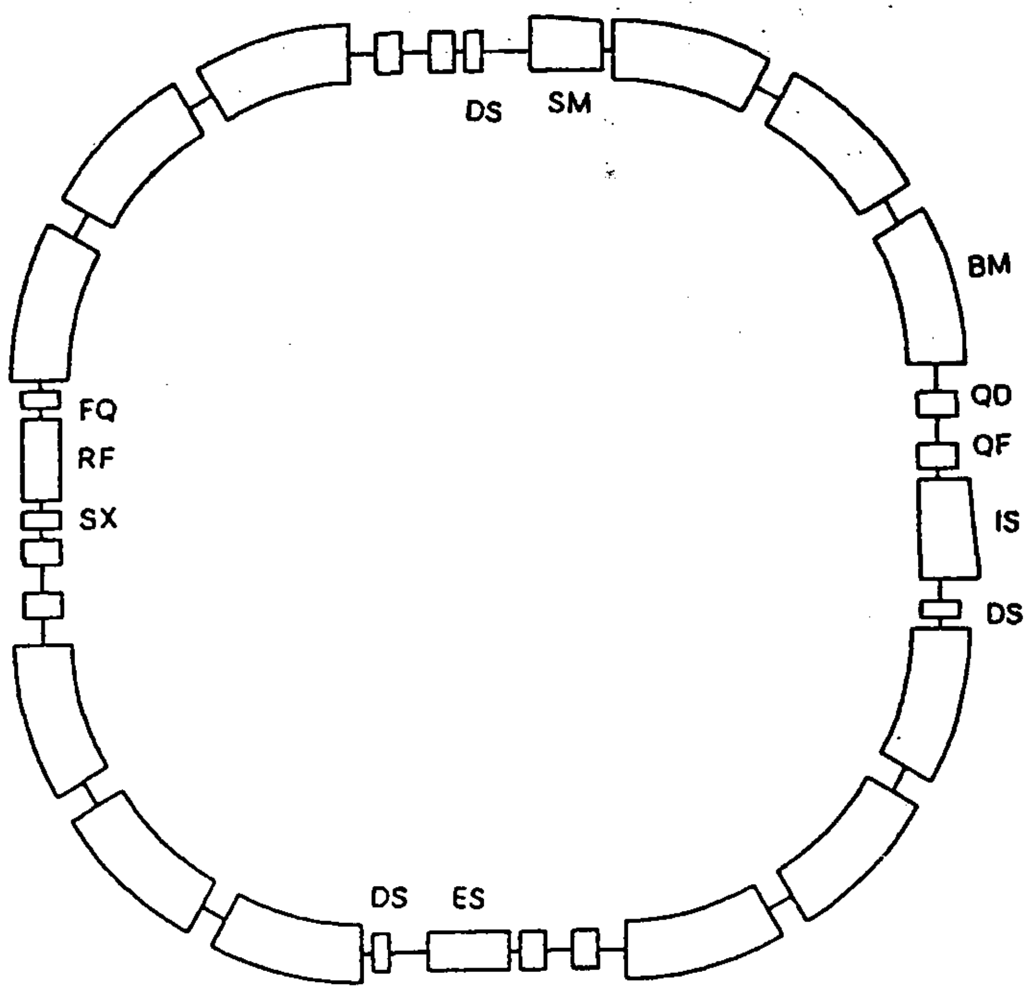


Fig.1.1: Layout of the 4-fold symmetry Medical Synchrotron.

QF Focusing Quadrupole. QD Defocusing Quadrupole. BM Bending Magnet. ES Electrostatic Septum. IS Injection Septum. SM Magnetic Septum. RF Radio Frequency. SX Sextupole. FQ Fast Quadrupole. DS Bumping Magnet.

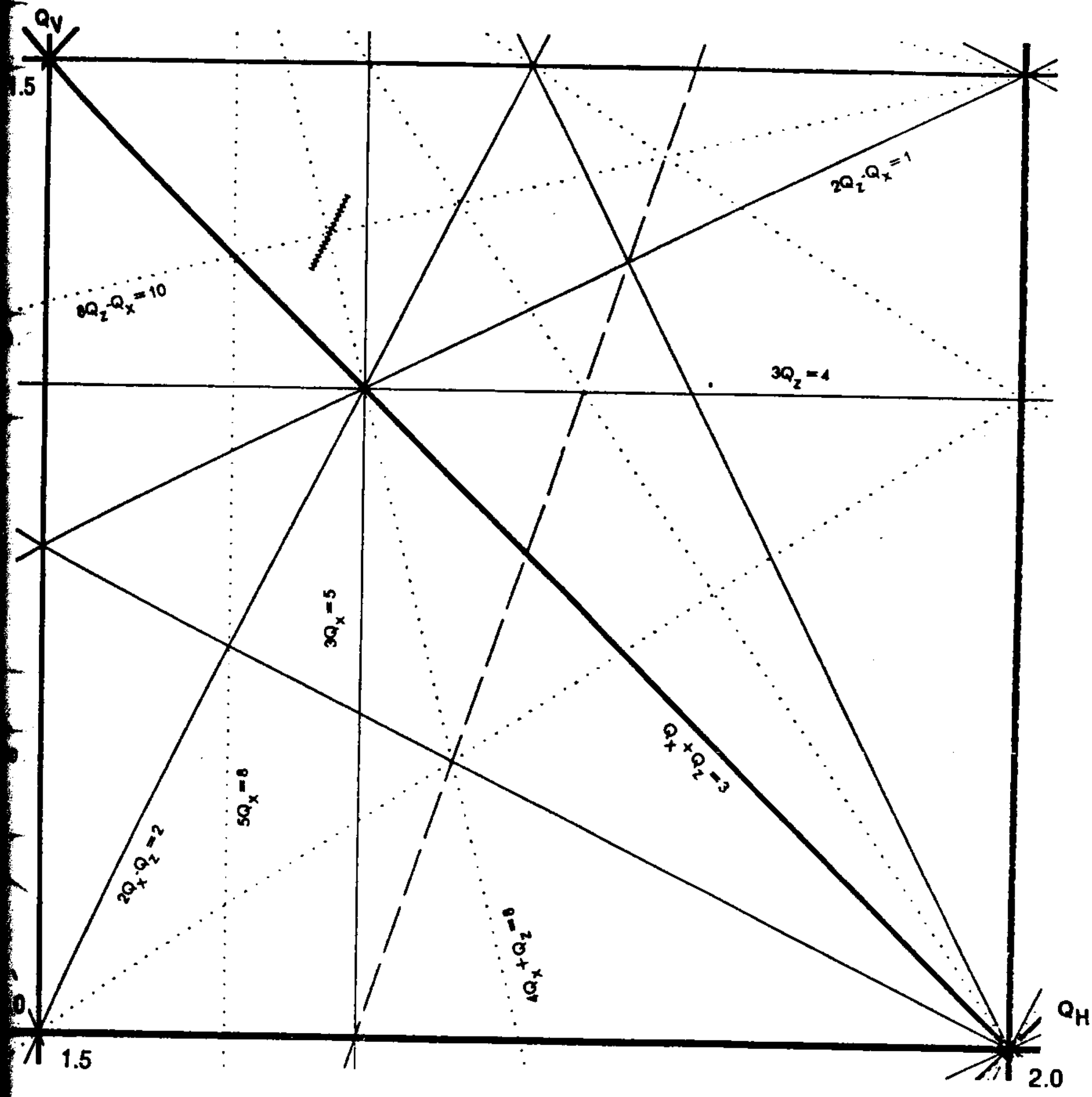


Fig. 1.2 Q_H - Q_V diagram for 4-fold lattice ($\Delta p/p$ from -0.008 to 0.008 step 0.001)

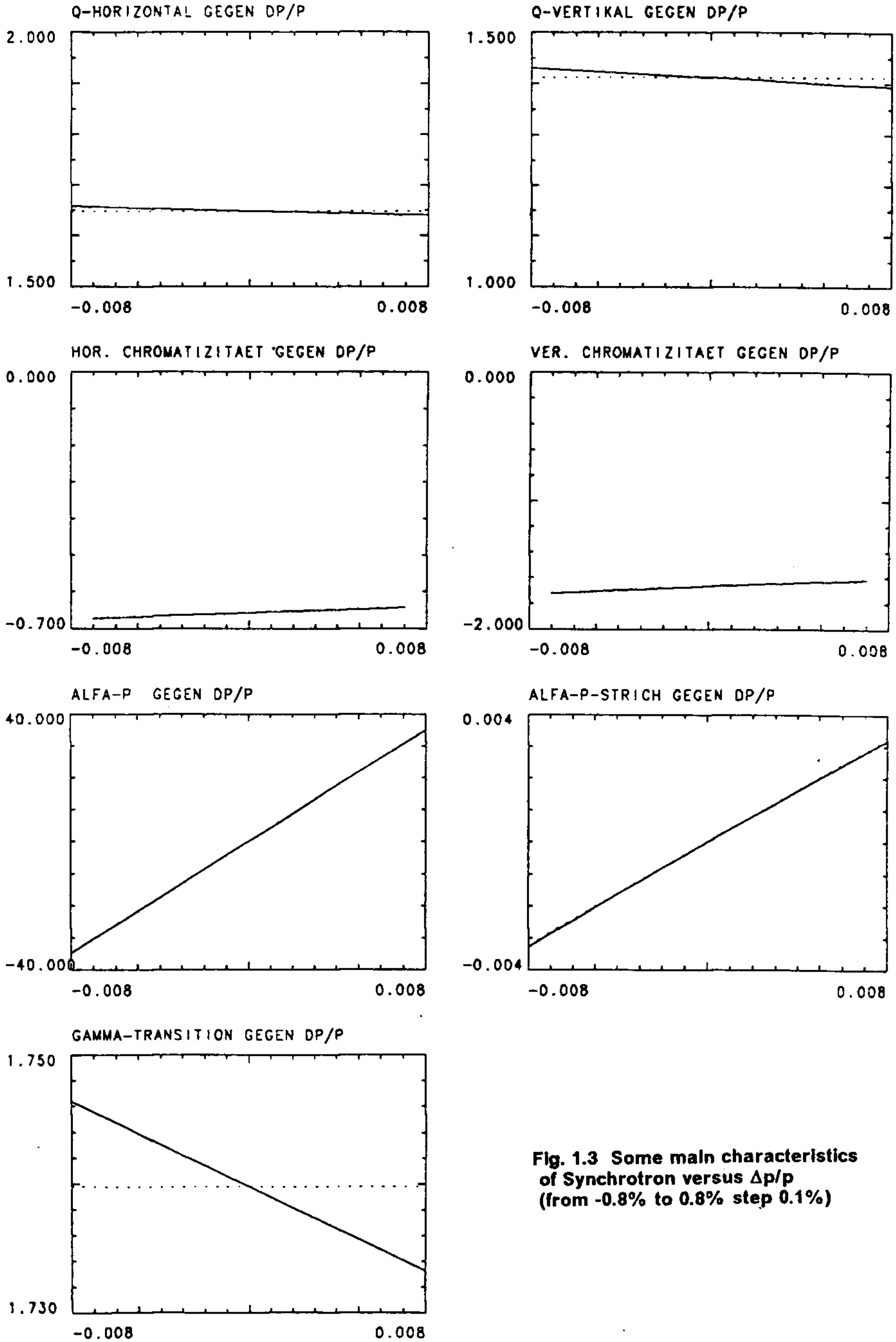


Fig. 1.3 Some main characteristics of Synchrotron versus $\Delta p/p$ (from -0.8% to 0.8% step 0.1%)

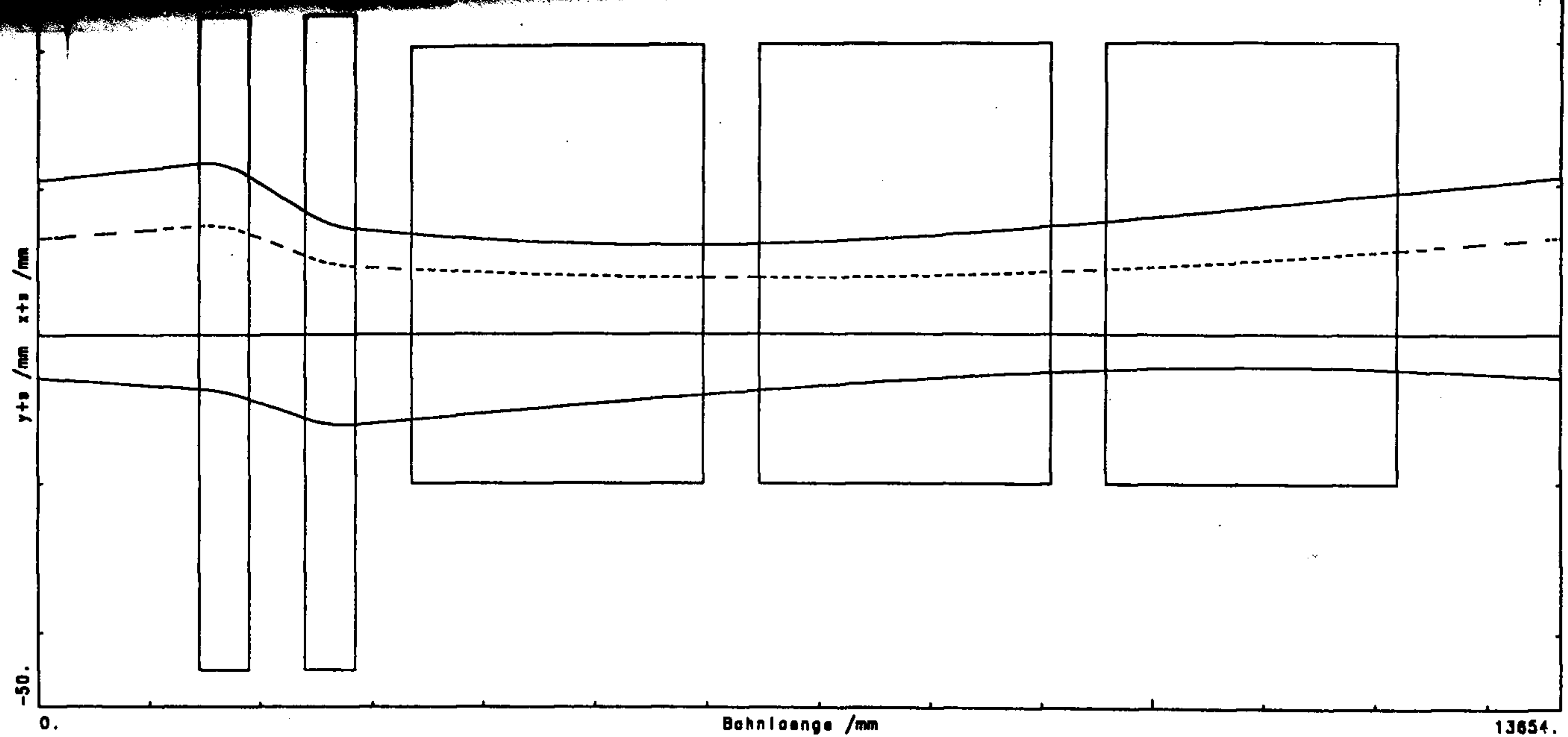


Fig. 1.4 Beam envelopes and dispersion function (dash line) for 4-fold lattice, plotted for $\varepsilon_h, \varepsilon_v = 6.225\ \pi$mm-mrad, $\Delta p/p = \pm 0.3\%$, horizontal and vertical plane.

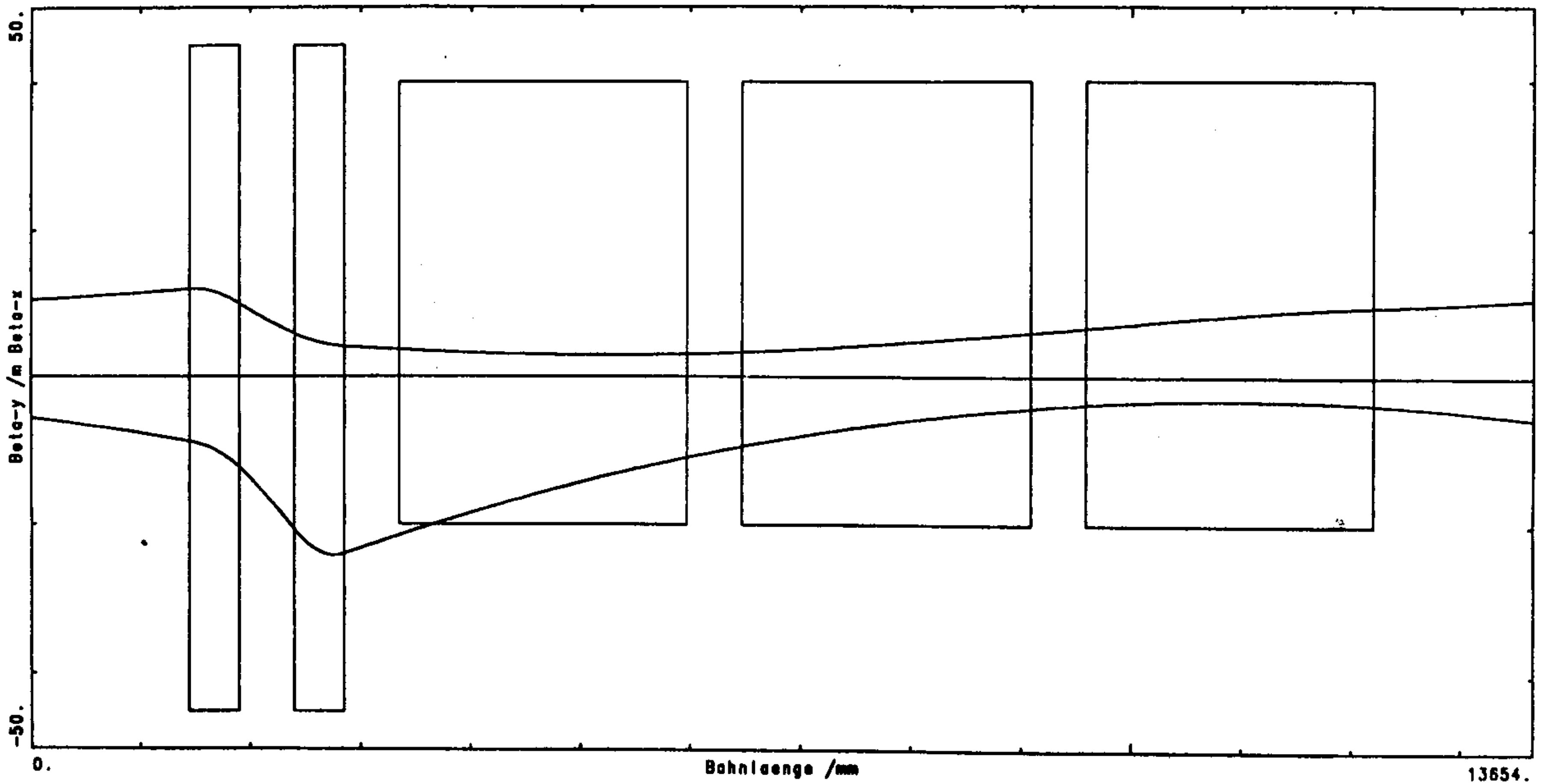


Fig. 1.5 β function for 4-fold structure.

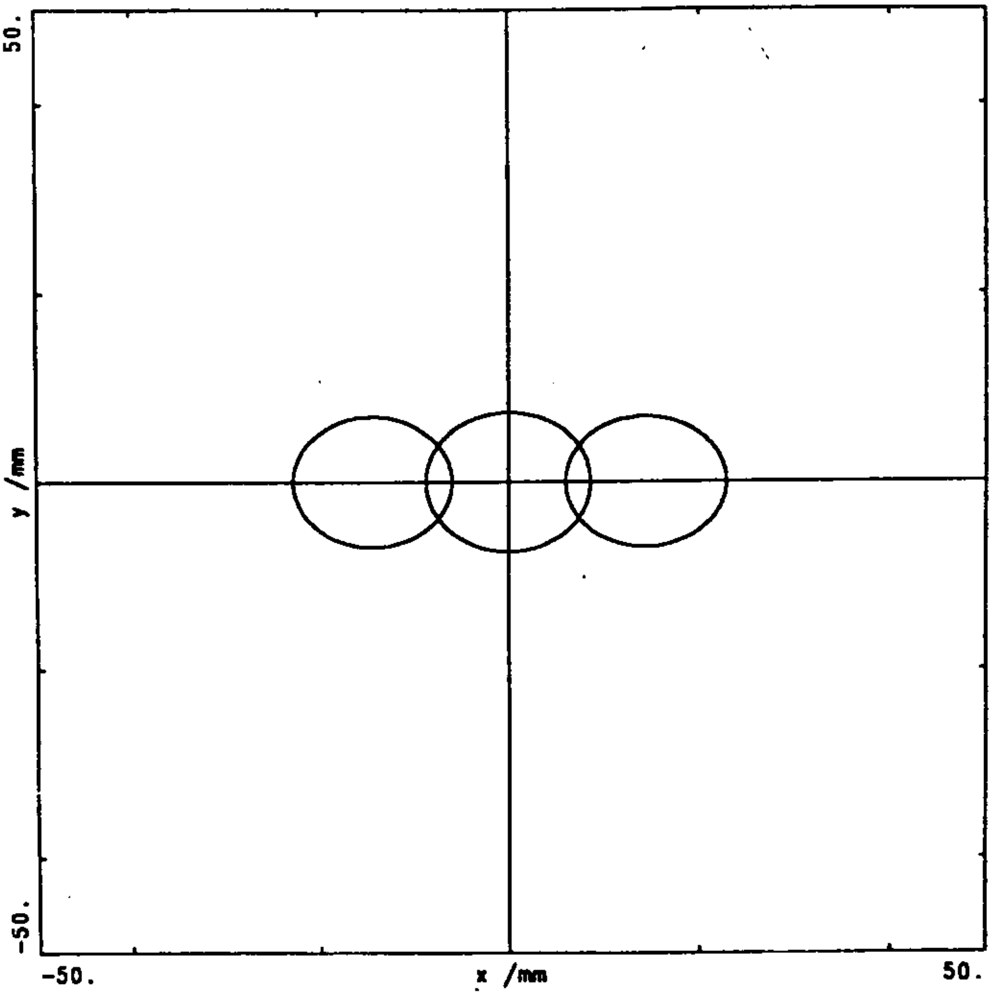


Fig. 1.6 Beam size at horizontal β maximum with ε_h, ε_v = 6.225 πmm-mrad and $\Delta p/p$ = $\pm$0.3% for 4-fold structure.

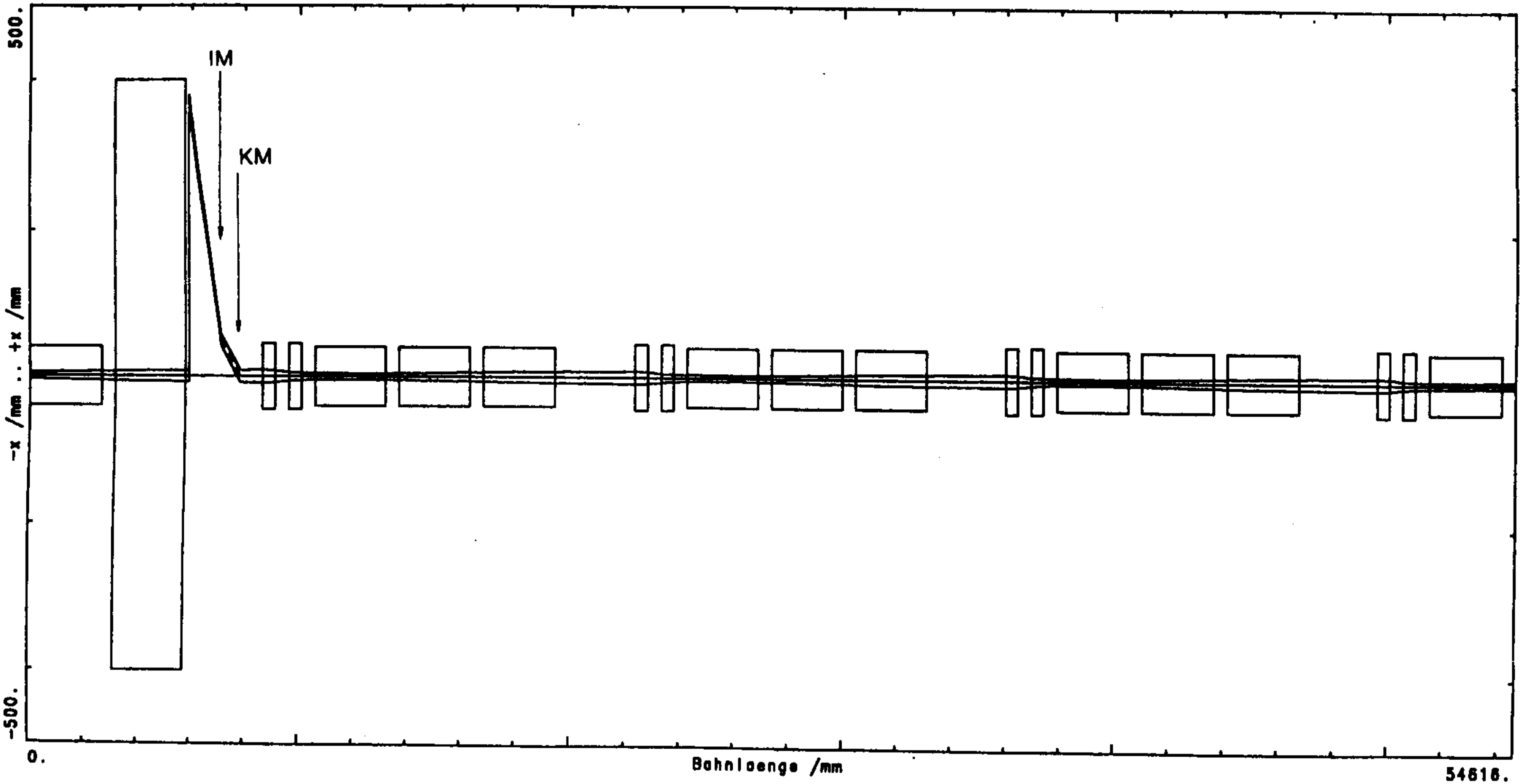


Fig. 1.7 Scheme of the single turn injection in 4-fold lattice lattice (ϵ_h = 6.225 πmm-mrad)
IM Inflector Magnet,
KM kicker Magnet.

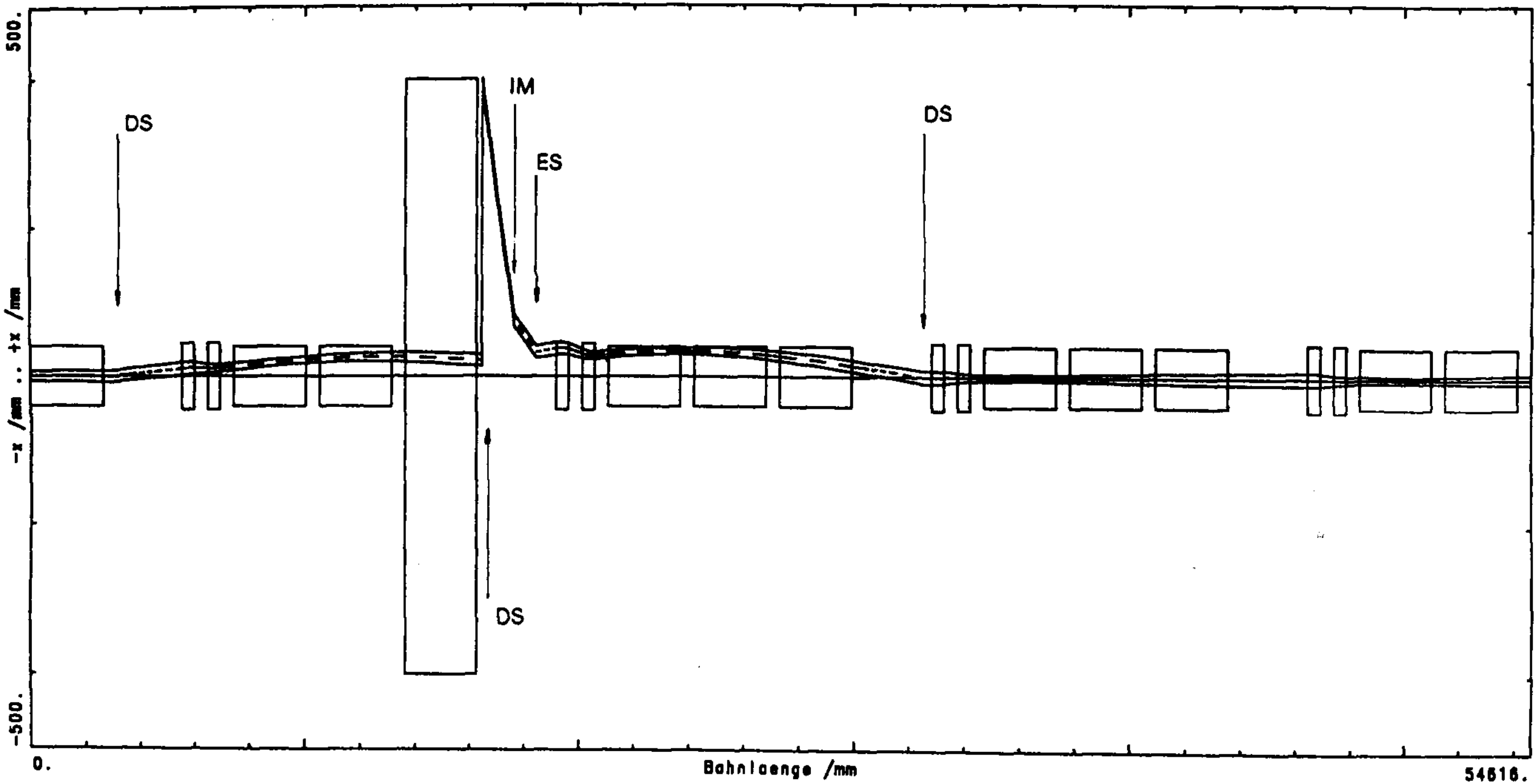


Fig. 1.8 Scheme of the Multiturn Injection into 4-fold structure (ε_h = 6.225 πmm-mrad)
IM Inflector Magnet,
ES Electrostatic Septum.
DS Bumping Magnet.

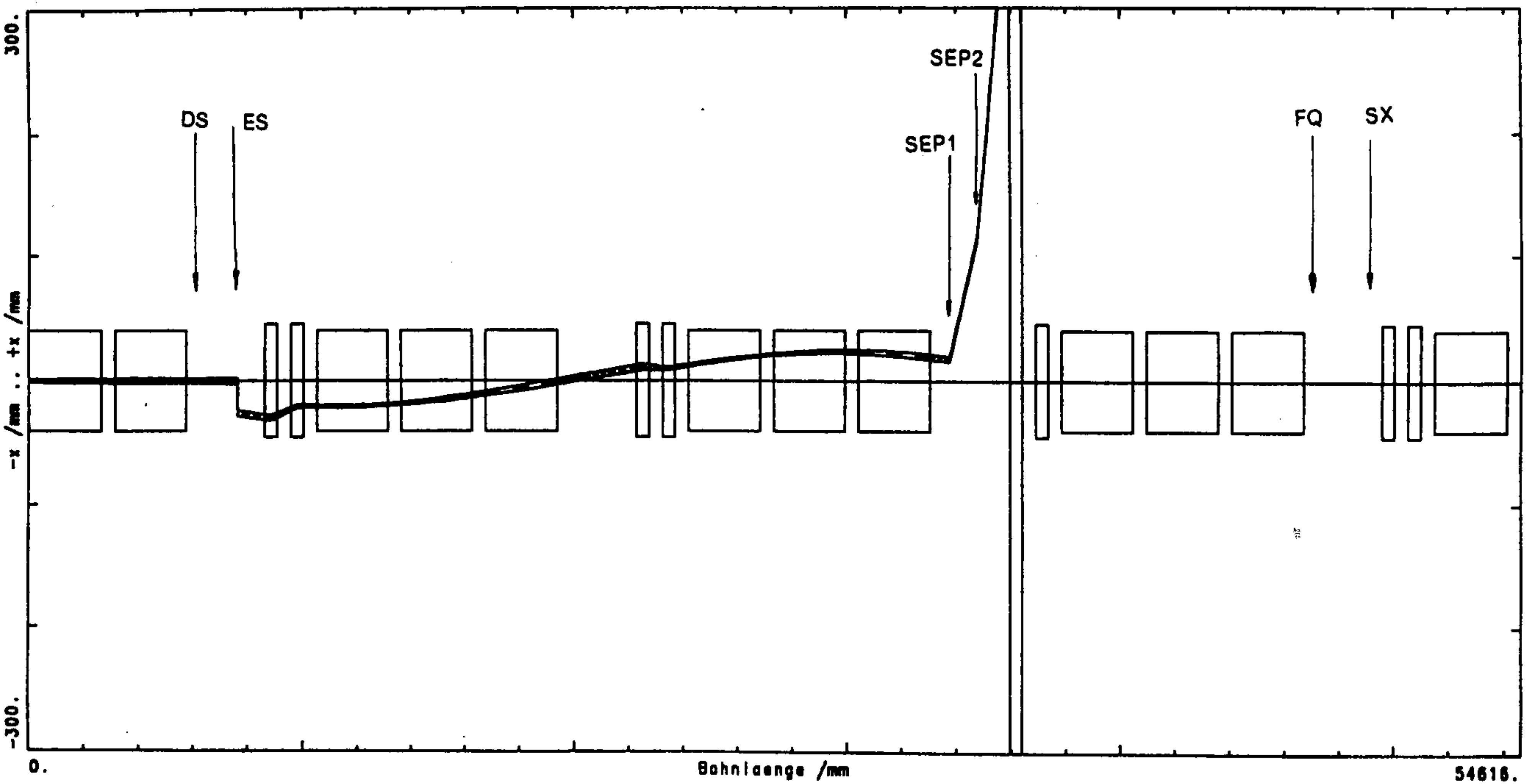


Fig. 1.9 Scheme of the resonance extraction at $Q_h = 1.67$ with sextupole excitation in 4-fold structure

SM Magnetic Septum,
ES Electrostatic Septum.
SX Sextupole.
FQ Fast Quadrupole.
DS Bumping Magnet.

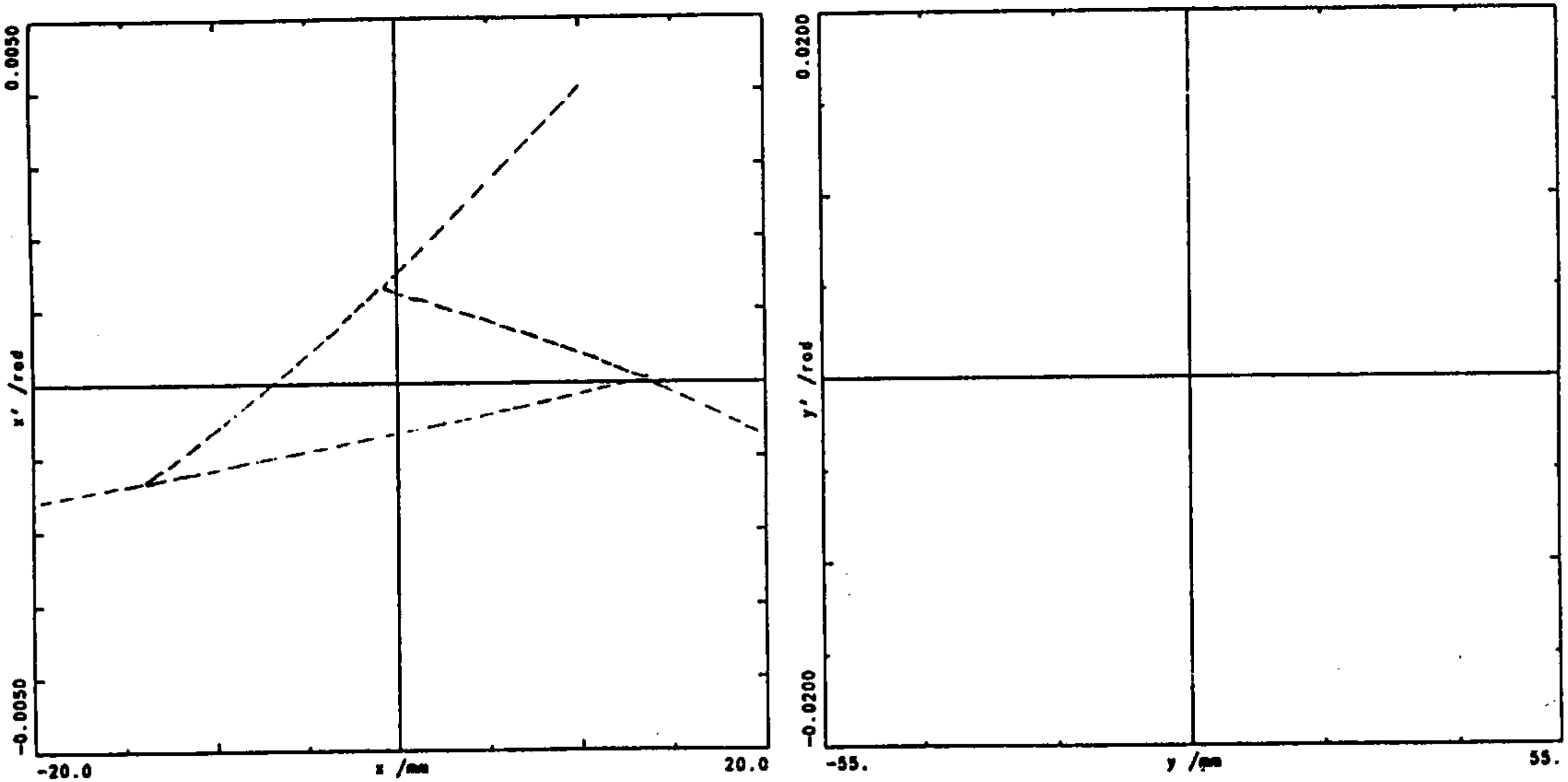


Fig. 1.10 Unstable fix points and separatrix in 4-fold structure at the electrostatic septum (Q_h=1.66)

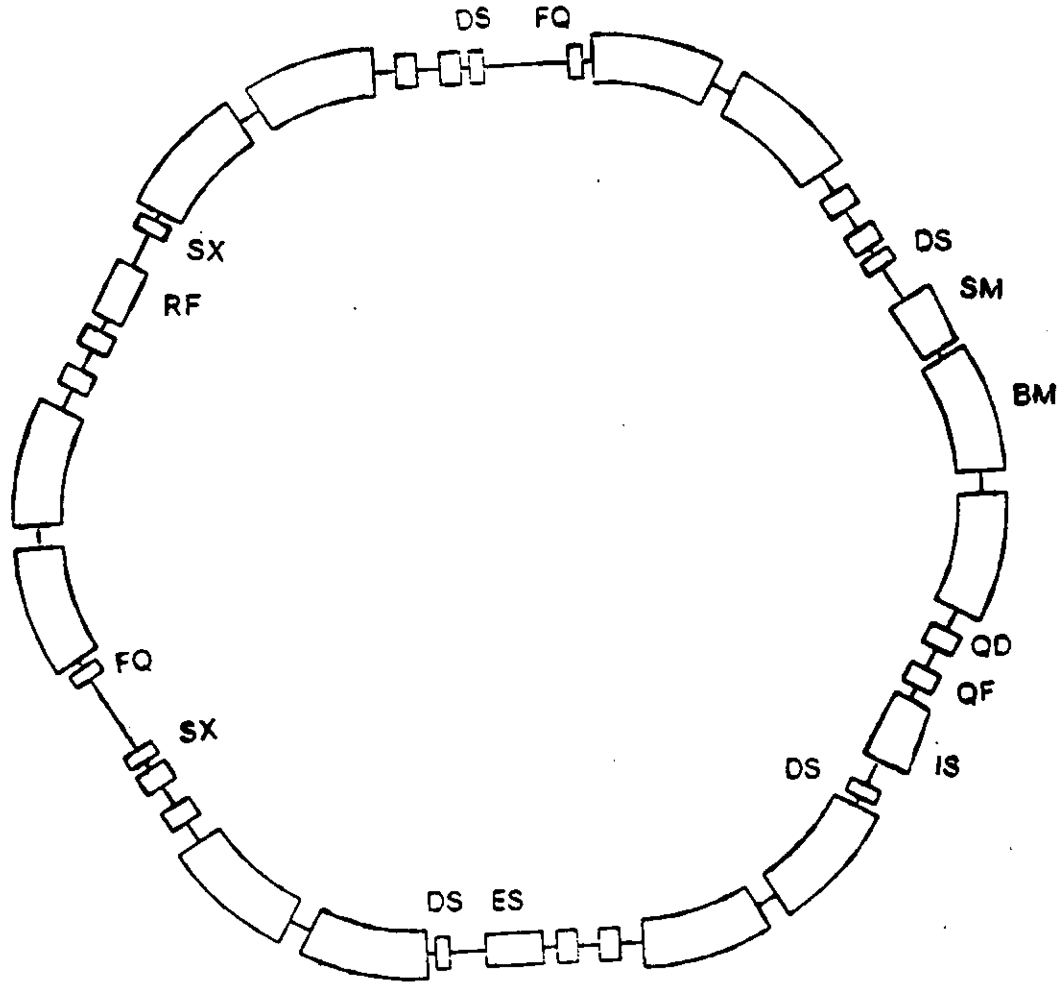


Fig.2.1: Layout of the 6-fold symmetry Medical Synchrotron.

QF Focusing Quadrupole. QD Defocusing Quadrupole. BM Bending Magnet. ES Electrostatic Septum. IS Injection Septum. SM Magnetic Septum. RF Radio Frequency. SX Sextupole. FQ Fast Quadrupole. DS Bumping Magnet.

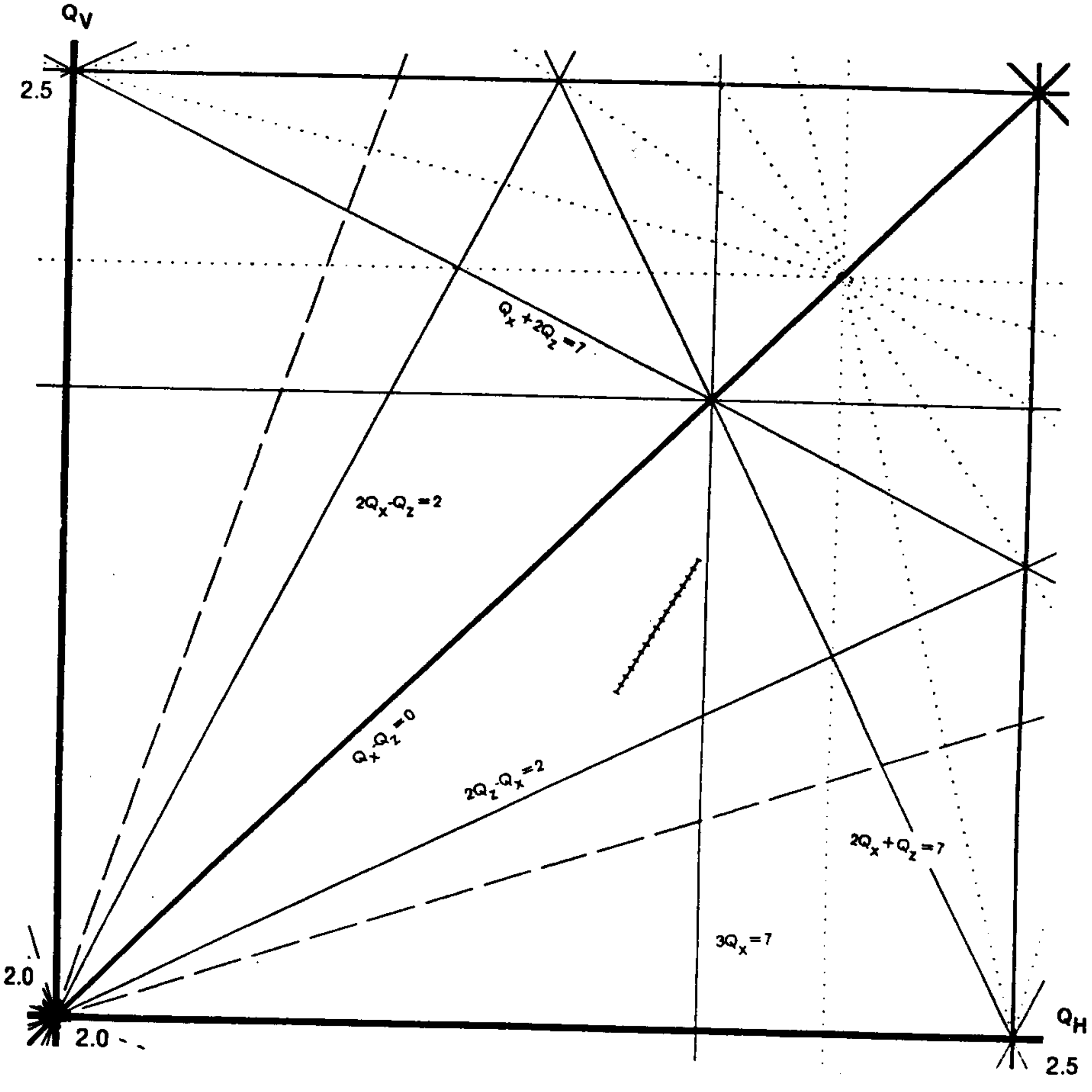
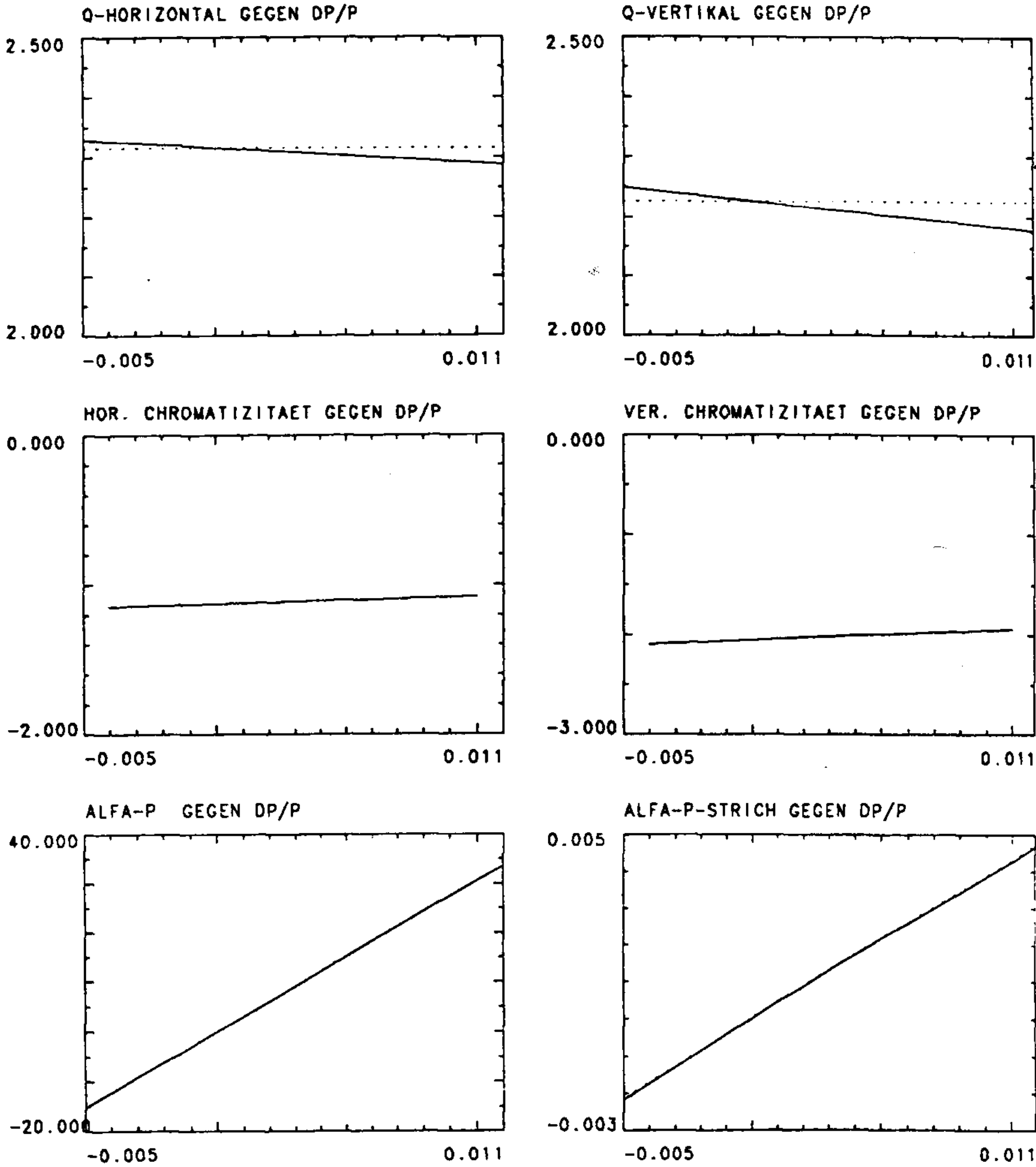


Fig. 2.2 Q_H - Q_V diagram for 6-fold lattice ($\Delta p/p$ from -0.005 to 0.0011 step 0.001)



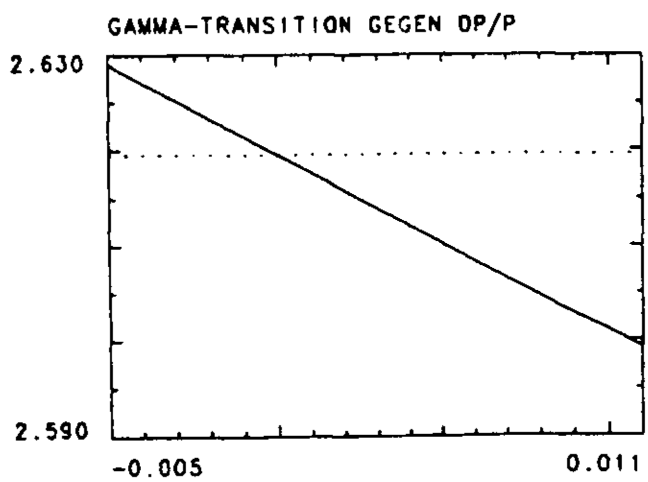


Fig. 2.3 Some main characteristics of Synchrotron versus Δp/p (from -0.5% to 0.11% step 0.1%)

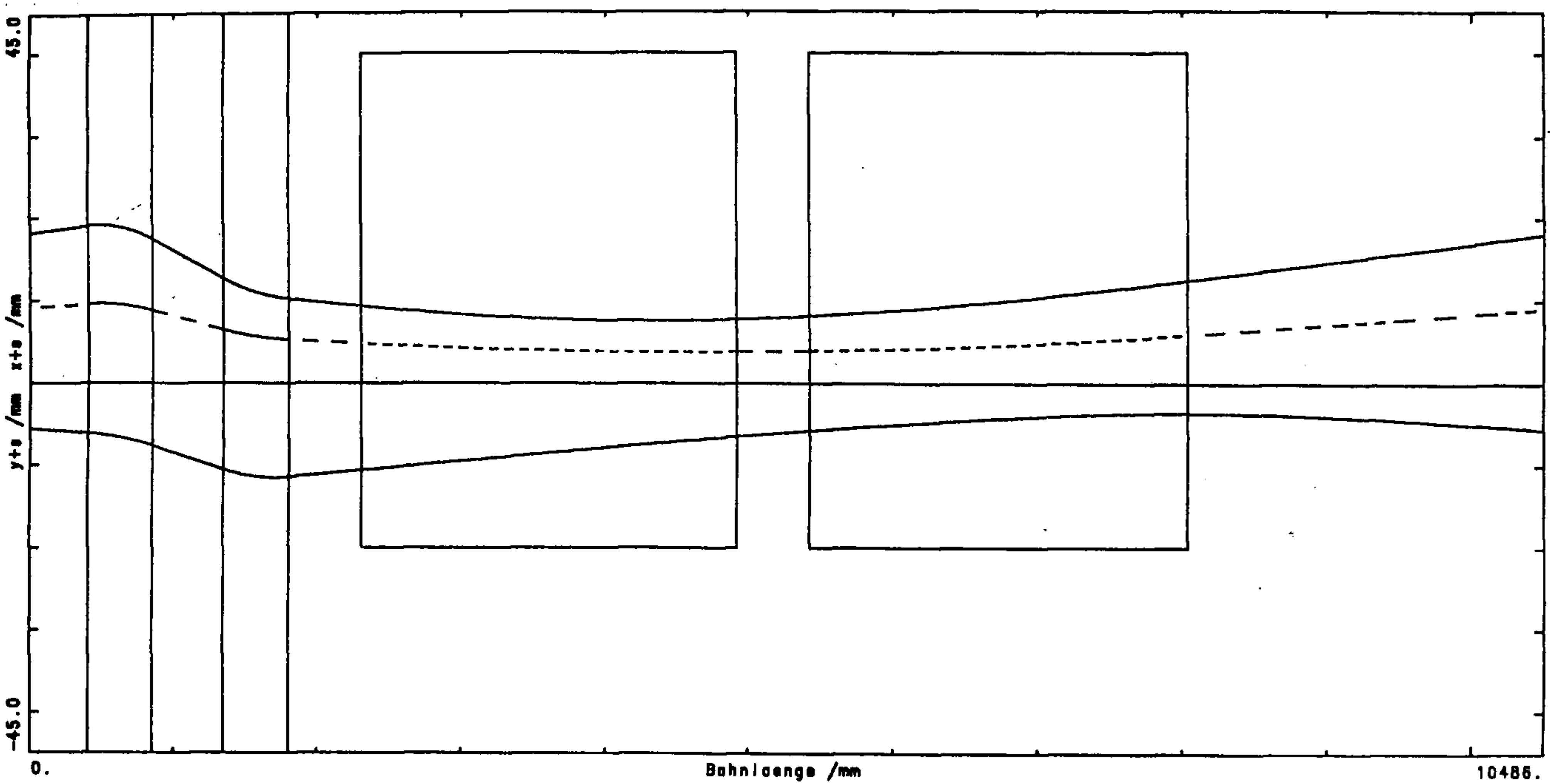


Fig. 2.4 Beam envelopes and dispersion function (dash line) for 6-fold lattice, plotted for $\varepsilon_h, \varepsilon_v = 6.225$ πmm-mrad, $\Delta p/p = \pm 0.3\%$, horizontal and vertical plane.

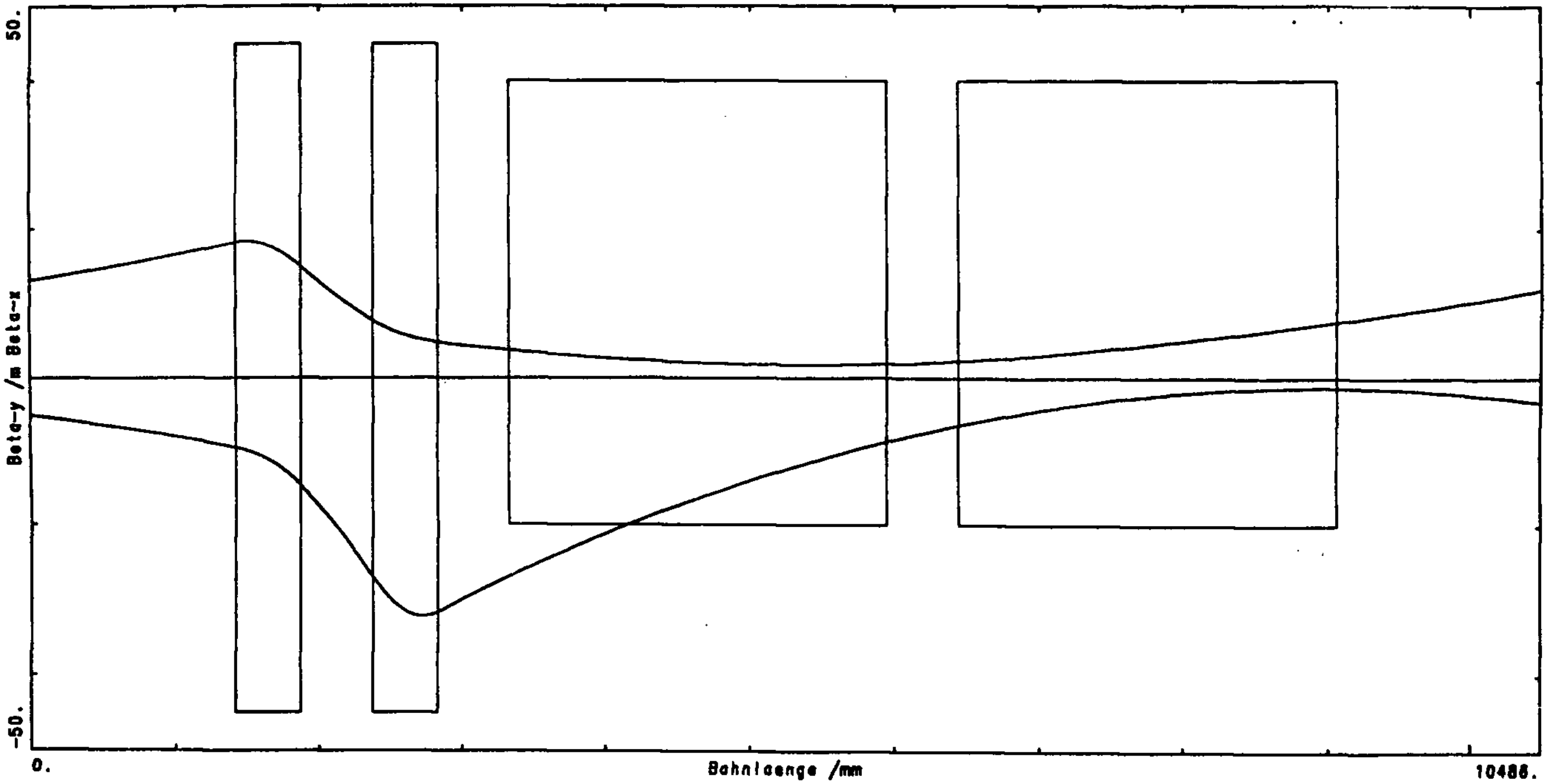


Fig. 2.5 β function for 6-fold structure.

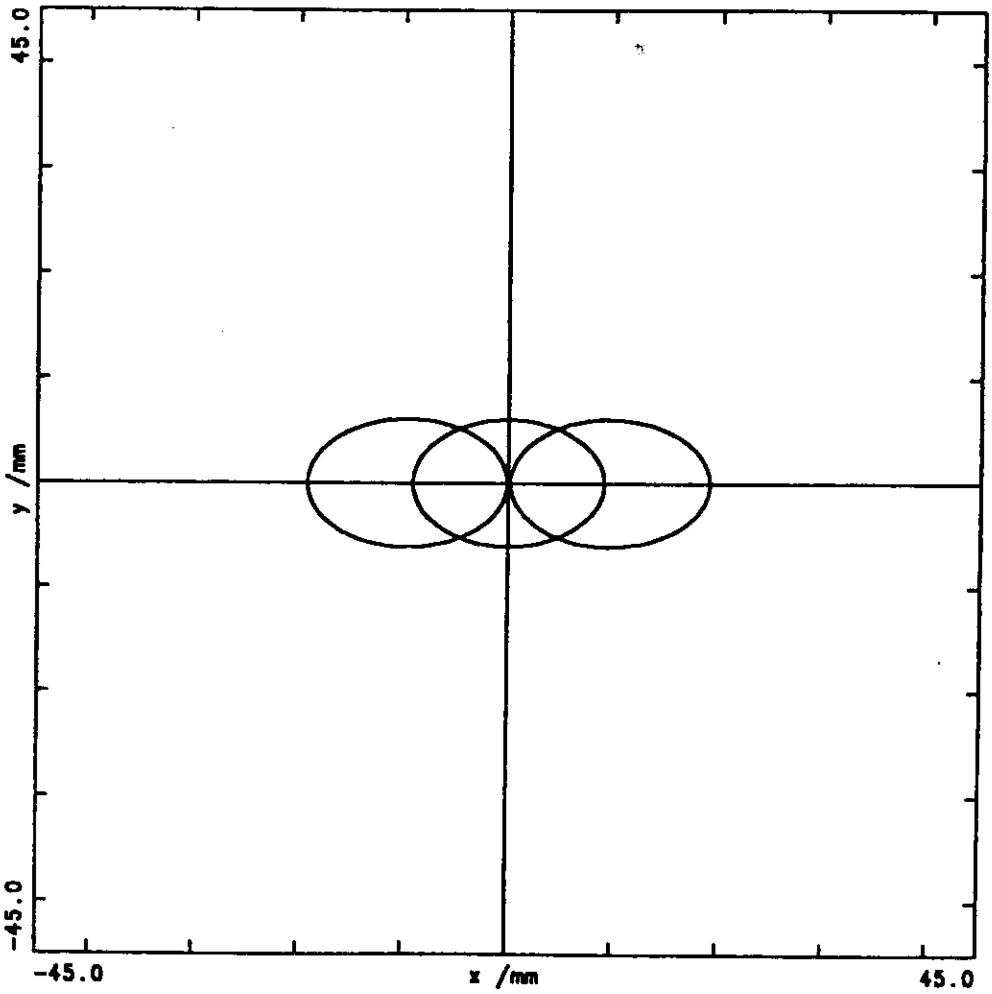


Fig. 2.6 Beam size at horizontal β maximum with ε_h, ε_v = 6.225 πmm-mrad and $\Delta p/p$ = ± 0.3% for 6-fold structure.

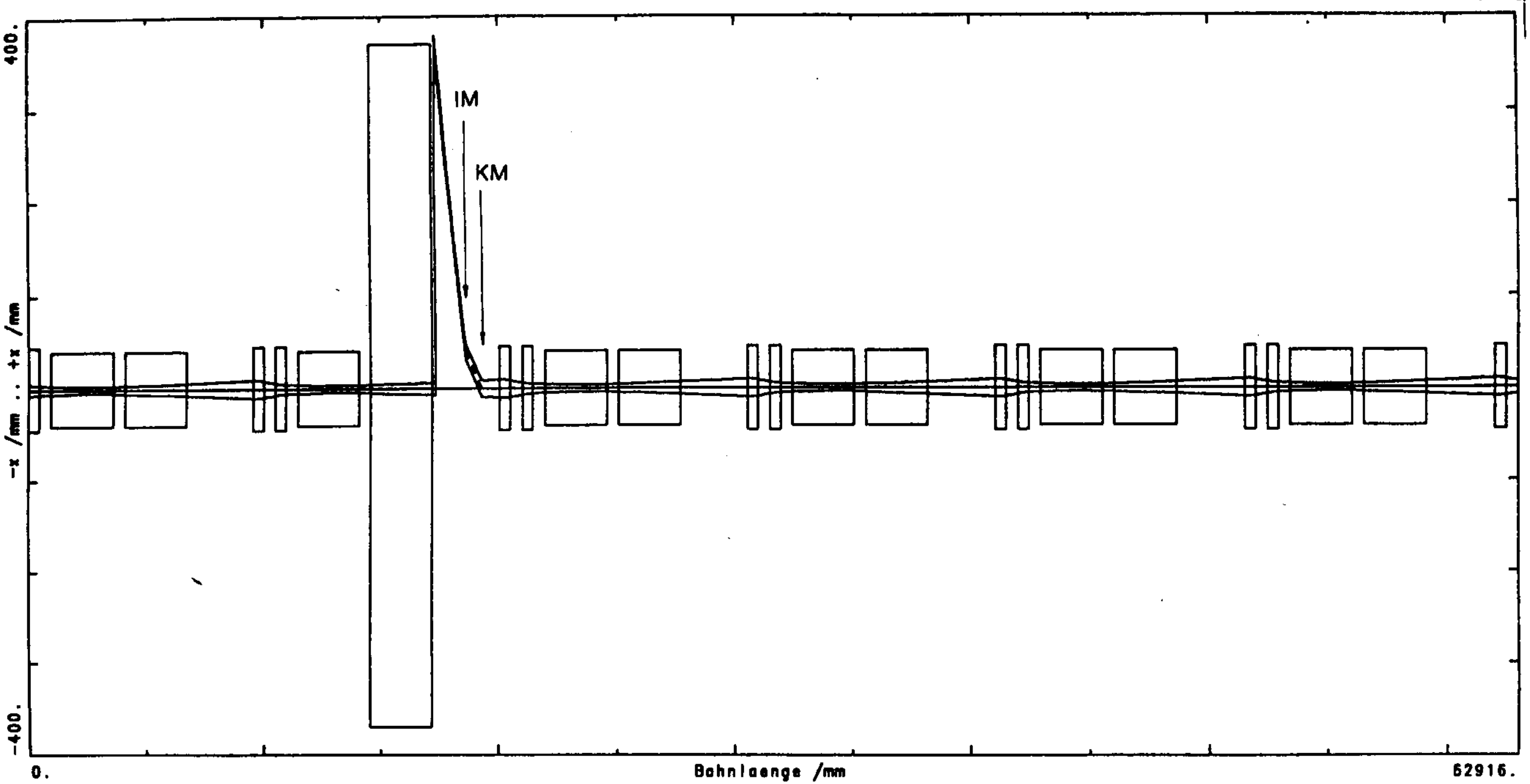


Fig. 2.7 Scheme of the single turn injection in 6-fold lattice (ϵ_h = 6.225 πmm-mrad)
IM Inflector Magnet,
KM kicker Magnet.

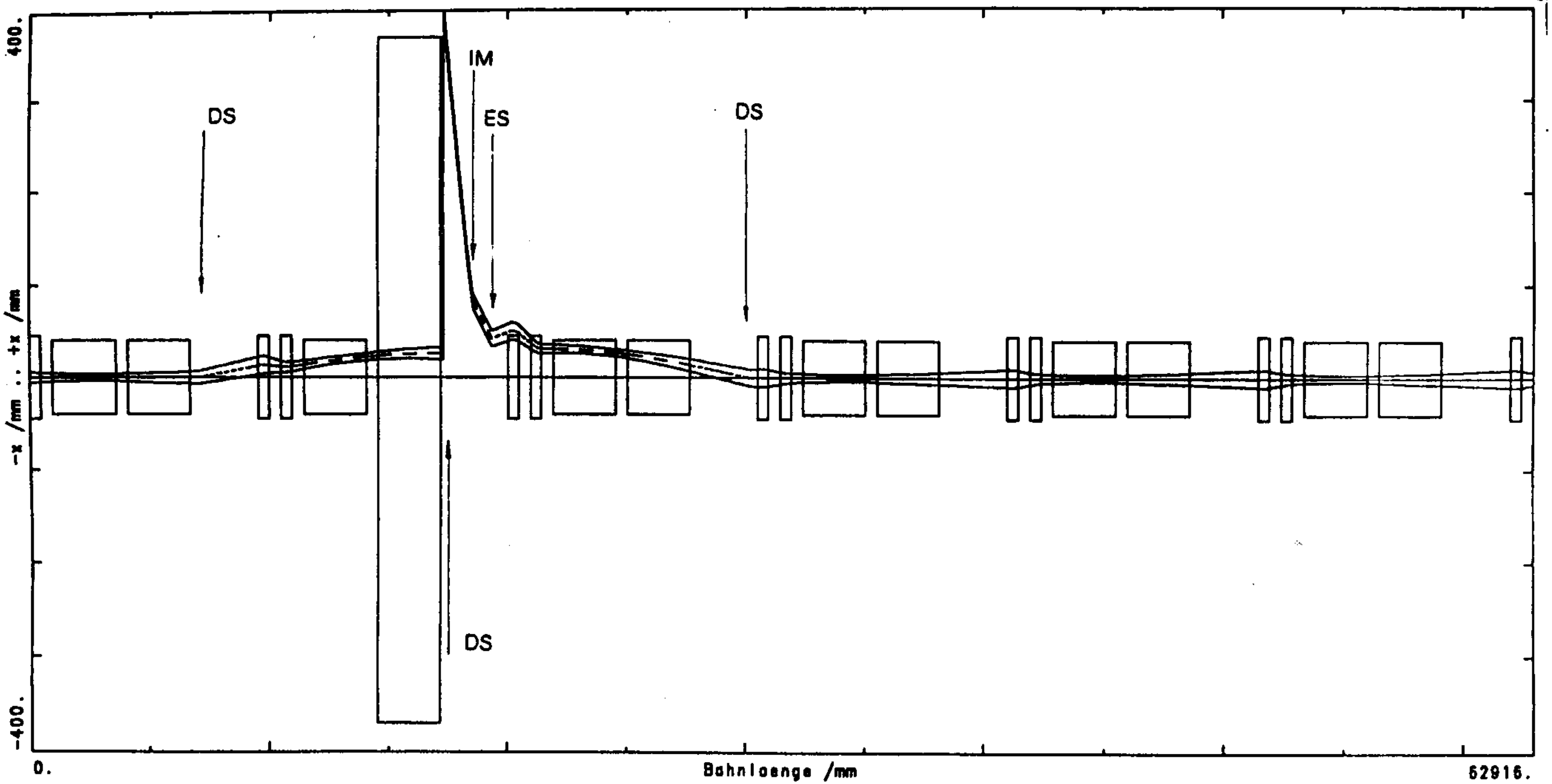


Fig. 2.8 Scheme of the Multiturn Injection into 6-fold structure (ε_h = 6.225 πmm-mrad)
IM Inflector Magnet,
ES Electrostatic Septum.
DS Bumping Magnet.

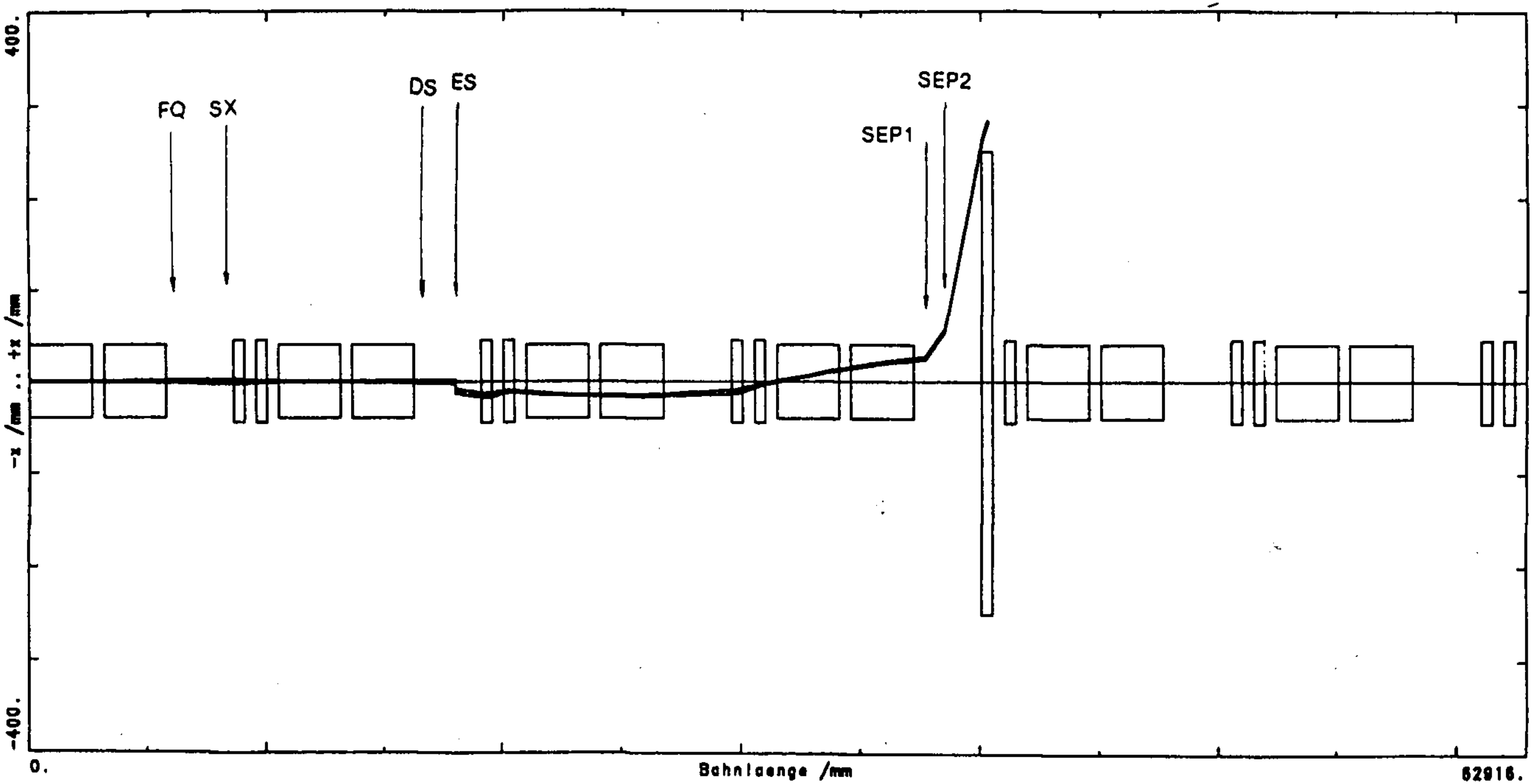


Fig. 2.9 Scheme of the resonance extraction at $Q_h = 2.33$ with sextupole excitation in 6-fold structure
SM Magnetic Septum,
ES Electrostatic Septum.
SX Sextupole.
FQ Fast Quadrupole.
DS Bumping Magnet.

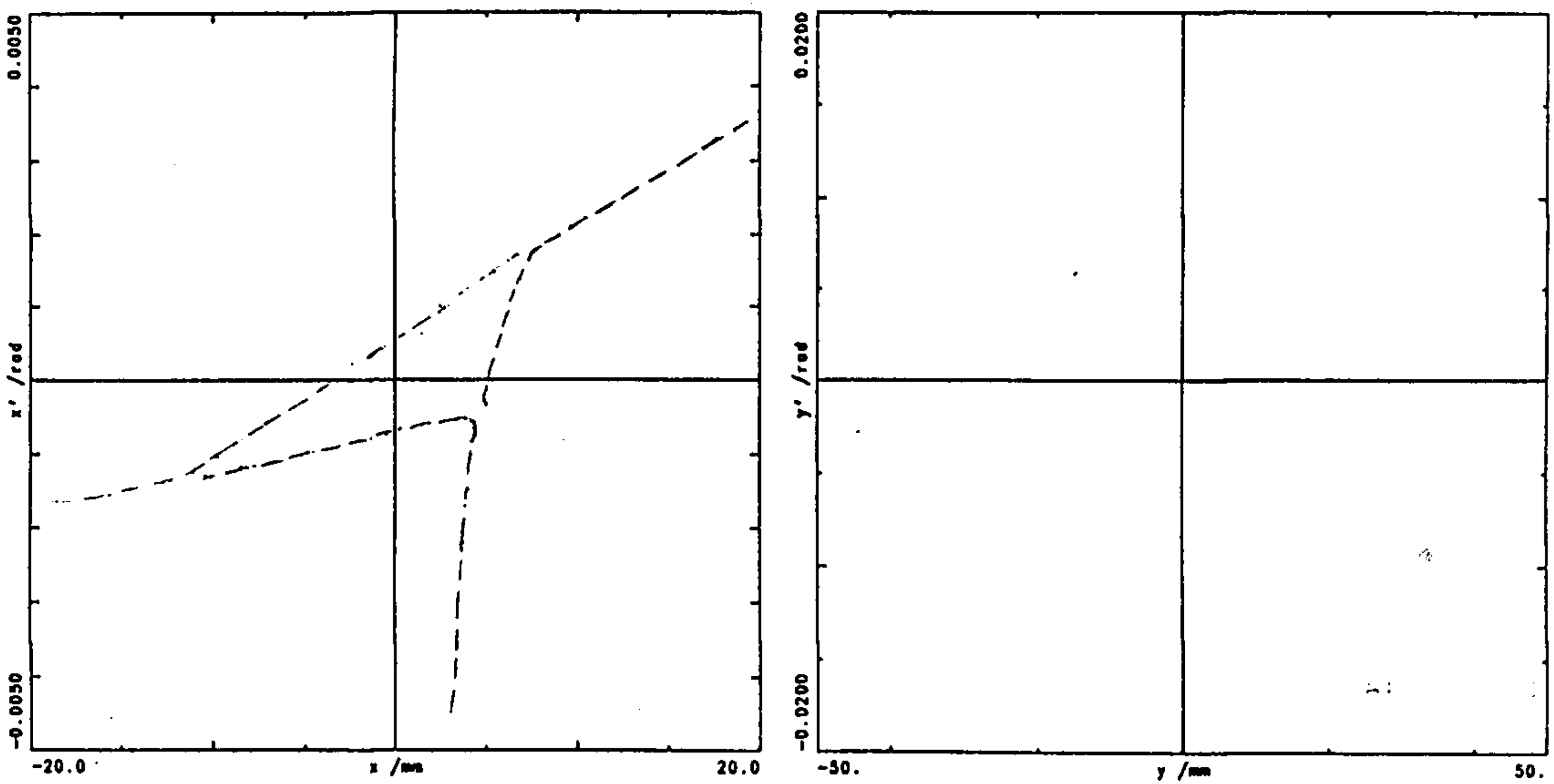


Fig. 2.10 Unstable fix points and separatrix in 6-fold structure at the electrostatic septum ($Q_h = 2.33$)